

## • Simplifying Improper Fractions

### Power Up

#### facts

Power Up I

#### mental math

- Time:** Rudy turns eleven years old today. How many months old is Rudy?
- Number Sense:** Reduce the fractions  $\frac{3}{6}$ ,  $\frac{3}{9}$ ,  $\frac{3}{12}$ , and  $\frac{3}{15}$ .
- Number Sense:**  $12\frac{1}{2} + 12\frac{1}{2}$
- Measurement:** Romy kicked the soccer ball 15 yards. How many feet is that?
- Powers/Roots:**  $1^3$
- Probability:** Lalo plans to flip a coin 10 times and record the results. Is it certain, likely, unlikely, or impossible that at least one flip will be heads?
- Calculation:**  $\sqrt{64}$ ,  $\times 3$ ,  $- 3$ ,  $\div 3$ ,  $\times 2$ ,  $- 2$ ,  $\div 2$ ,  $\div 3$ ,  $\div 2$
- Roman Numerals:**<sup>1</sup> Write XII in our number system.

#### problem solving

Choose an appropriate problem-solving strategy to solve this problem. This table lists the years from 2009 to 2014 and the day of the week on which each year begins. Notice that each year begins one day of the week later than the first day of the previous year until 2013. Since 2012 is a leap year and has an additional day, the year 2013 begins an additional day later. Copy this table and continue it through the year 2023, which begins on a Sunday.

Year	First Day
2009	Thursday
2010	Friday
2011	Saturday
2012	Sunday
2013	Tuesday
2014	Wednesday

<sup>1</sup> In Lessons 91–105, the Mental Math section “Roman Numerals” reviews concepts from Appendix Topic A. You may skip these Mental Math problems if you have not covered Appendix Topic A.

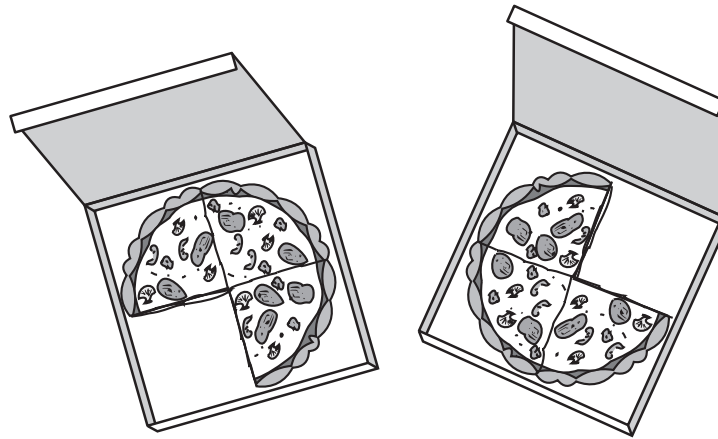
## New Concept

### Math Language

When a fraction has a numerator that is equal to or greater than the denominator, the fraction is called an *improper fraction*.

We have learned two ways to simplify fractions. We have converted improper fractions to whole numbers or mixed numbers, and we have reduced fractions. In some cases we need to use both ways to simplify a fraction. Consider the following story:

*After the party, some pizza was left over. There was  $\frac{3}{4}$  of a pizza in one box and  $\frac{3}{4}$  of a pizza in another box. Altogether, how much pizza was in the two boxes?*



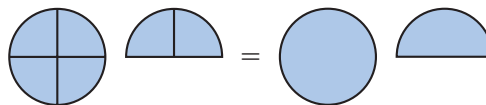
In this story about combining, we add  $\frac{3}{4}$  to  $\frac{3}{4}$ .

$$\frac{3}{4} + \frac{3}{4} = \frac{6}{4}$$

We see that the sum is an improper fraction. To convert an improper fraction to a mixed number, we divide the numerator by the denominator and write the remainder as a fraction.

$$\frac{6}{4} \rightarrow 4 \overline{)6} \begin{array}{r} 1\frac{2}{4} \\ 4 \\ \hline 2 \end{array}$$

The improper fraction  $\frac{6}{4}$  is equal to the mixed number  $1\frac{2}{4}$ . However,  $1\frac{2}{4}$  can be reduced.



$$1\frac{2}{4} = 1\frac{1}{2}$$

The simplified answer to  $\frac{3}{4} + \frac{3}{4}$  is  $1\frac{1}{2}$ .

### Math Language

When we write a quotient as a fraction or a mixed number, the remainder is the numerator of a fraction that has the divisor as its denominator.

**Discuss** Use fractions to explain why  $1\frac{2}{4} = \frac{6}{4}$ .

### Example 1

Write  $\frac{8}{6}$  as a mixed number in lowest terms.

To convert  $\frac{8}{6}$  to a mixed number, we divide 8 by 6 and get  $1\frac{2}{6}$ . Then we reduce  $1\frac{2}{6}$  by dividing both terms of the fraction by 2 and get  $1\frac{1}{3}$ .

$$\begin{array}{ccc} \text{Convert} & & \text{Reduce} \\ \frac{8}{6} = 1\frac{2}{6} & \longrightarrow & 1\frac{2}{6} = 1\frac{1}{3} \end{array}$$

**Verify** Use fractions to explain why  $1\frac{2}{6} = \frac{8}{6}$ .

### Example 2

The dictionary is  $1\frac{7}{8}$  in. thick, and the thesaurus is  $1\frac{3}{8}$  in. thick. If the two books are side by side, how thick are they altogether?

We add  $1\frac{7}{8}$  and  $1\frac{3}{8}$  to get  $2\frac{10}{8}$ . We convert the improper fraction  $\frac{10}{8}$  to  $1\frac{2}{8}$  and add it to the 2 to get  $3\frac{2}{8}$ . Finally, we reduce the fraction to get  $3\frac{1}{4}$ .

$$\begin{array}{ccccc} & \text{Add} & & \text{Convert} & & \text{Reduce} \\ 1\frac{7}{8} + 1\frac{3}{8} = 2\frac{10}{8} & \longrightarrow & 2\frac{10}{8} = 3\frac{2}{8} & \longrightarrow & 3\frac{2}{8} = 3\frac{1}{4} \end{array}$$

**Justify** Use fractions to explain why  $2\frac{10}{8} = 3\frac{2}{8}$ .

## Activity

### Modeling Improper Fractions

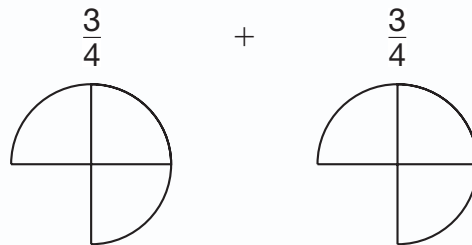
Materials needed:

- fraction manipulatives saved from Investigations 2 and 3

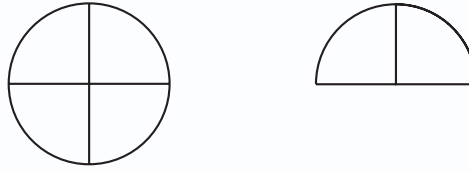
Use your fraction manipulatives to model each problem below.

Write the mixed number or whole-number answer for each

problem. For example, model  $\frac{3}{4} + \frac{3}{4}$  this way:



Then combine the pieces to show that  $\frac{3}{4} + \frac{3}{4} = 1\frac{1}{2}$ .



Model:

1.  $\frac{1}{2} + \frac{1}{2} + \frac{1}{2}$

2.  $\frac{3}{4} + \frac{3}{4}$

3.  $\frac{5}{8} + \frac{5}{8}$

4.  $\frac{2}{3} + \frac{2}{3}$

### Lesson Practice

Simplify each fraction or mixed number:

a.  $\frac{6}{4}$

b.  $\frac{10}{6}$

c.  $2\frac{8}{6}$

d.  $3\frac{10}{4}$

e.  $\frac{10}{4}$

f.  $\frac{12}{8}$

g.  $4\frac{14}{8}$

h.  $1\frac{10}{8}$

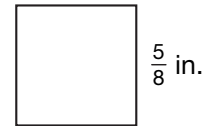
Perform each indicated operation. Simplify your answers. You may use fraction manipulatives to solve i and j.

i.  $1\frac{5}{6} + 1\frac{5}{6}$

j.  $2\frac{3}{4} + 4\frac{3}{4}$

k.  $\frac{5}{3} \times \frac{3}{2}$


- l. **Connect** Each side of this square is  $\frac{5}{8}$  inches long. What is the perimeter of the square? Show your work.



### Written Practice

*Distributed and Integrated*

- \*1. Two fathoms deep is 12 feet deep. How deep is 10 fathoms?  
(49)

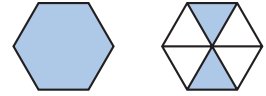
2.  **Explain** When Jessica babysits, she is paid \$6.50 per hour. If she babysits on Saturday from 10:30 a.m. to 3:30 p.m., how much money will she be paid? Explain how you found your answer.  
(49)

- \*3. **Represent** Use digits to write the number one hundred fifty-four million, three hundred forty-three thousand, five hundred fifteen.  
(52)

- \*4. a. How many quarter-mile laps does Tyler have to run to complete 1 mile?  
(87)
- b. How many quarter-mile laps does Tyler have to run to complete 5 miles?

5. **Analyze** Write a fraction equal to  $\frac{3}{4}$  that has a denominator of 8. Add that fraction to  $\frac{5}{8}$ . Remember to convert the answer to a mixed number.

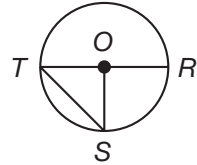
6. What mixed number names the number of shaded hexagons?



\*7. **Multiple Choice** Which segment does *not* name a radius of this circle?

A  $\overline{SO}$   
C  $\overline{TS}$

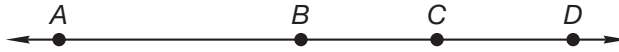
B  $\overline{OR}$   
D  $\overline{OT}$



8. Compare:  $\frac{1}{2}$  of 2  $\bigcirc$   $2 \times \frac{1}{2}$

\*9. A pentagonal prism has how many vertices?

10.  $AB$  is 3.2 cm.  $BC$  is 1.8 cm.  $CD$  equals  $BC$ . Find  $AD$ .



\*11.  $1\frac{3}{4} + 1\frac{3}{4}$

\*12.  $5\frac{7}{8} - 1\frac{3}{8}$

\*13.  $3 \times \frac{3}{8}$

14.  $\$10 - (\$1.25 + 35\text{¢})$

15.  $\begin{array}{r} \$4.32 \\ \times \quad 5 \\ \hline \end{array}$

16.  $\begin{array}{r} 416 \\ \times 740 \\ \hline \end{array}$

17.  $4.51 - (2.3 + 0.65)$

18.  $960 \div 8$

19.  $80 \overline{)9600}$

20.  $5m = \$12.00$


\*21.  $\frac{5}{2} \times \frac{2}{3}$

\*22.  $\frac{2}{3} \div \frac{1}{3}$

\*23.  $\frac{2}{3} \div \frac{1}{6}$

- \*24. <sup>(57)</sup> If two number cubes are rolled, the sum of the two top numbers can be any number from 2 through 12. Since there are six ways the first cube can land and six ways the second cube can land, there are 36 possible combinations. The table below shows the number of combinations there are for each sum. For example, three combinations equal a total of 10. They are  $4 + 6$ ,  $6 + 4$ , and  $5 + 5$ . Refer to the table below to answer parts **a–c**.

<b>Sum of Numbers</b>	2	3	4	5	6	7	8	9	10	11	12
<b>Number of Ways</b>	1	2	3	4	5	6	5	4	3	2	1

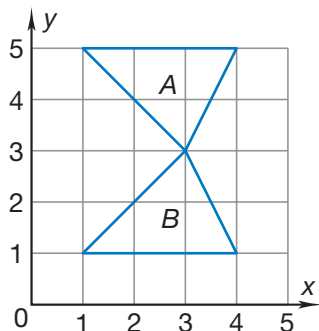
- How many combinations total six? List them.
- What is the probability of rolling a sum of 7 with one toss of two number cubes?
-  **Predict** If two number cubes are rolled once, which outcome is more likely: a sum of 4 or a sum of 9? Explain.


25. <sup>(Inv. 4)</sup> **Conclude** Assuming that the sequence below repeats with period 3, write the next 5 terms:

4, 4, 1, 4, 4, ...

- \*26. <sup>(84)</sup> The days of the week are Sunday, Monday, Tuesday, Wednesday, Thursday, Friday, and Saturday. Make a list of the number of letters in each name. Friday, for instance, has 6 letters and Saturday has 8. Refer to your list of numbers to answer parts **a–d**.
- What number is the median?
  - What number is the mode?
  - What is the range?
  - Find the mean and write it as a mixed number.

- \*27. Multiple Choice** Which transformation would move triangle *A* to the position of triangle *B*?  
*(Inv. 8)*



- A** translation      **B** rotation      **C** reflection      **D** slide
- 28.** On a November day, the low temperature in Minneapolis, Minnesota, was  $19^{\circ}\text{F}$ . The high temperature was  $34^{\circ}\text{F}$ . What was the range of temperatures that day in Minneapolis?  
*(27)*
- 29.** Yesterday it took Lucius  $\frac{1}{4}$  of an hour to walk to school and  $\frac{1}{4}$  of an hour to walk home from school. In simplest form, what fraction of one hour did Lucius spend walking to and from school yesterday?  
*(41)*
- \*30.**  **Explain** A square classroom at Charles School is 784 square feet. What is the length of each side of the room?  
*(78)*

**Early Finishers**  
*Real-World Connection*

Dala and Tessa just finished decorating their room. They have  $\frac{3}{4}$  of a gallon of paint left in one bucket and  $\frac{1}{2}$  of a gallon of paint in another bucket.

- Use your fraction manipulatives to find out how much paint they have altogether.
- They also used several rolls of wallpaper. They have  $\frac{15}{8}$  rolls left. Write this number as a mixed number.

## • Dividing by Two-Digit Numbers

### Power Up

#### facts

#### mental math

#### problem solving

#### Power Up I

- a. **Number Sense:** Reduce the fractions  $\frac{6}{8}$ ,  $\frac{6}{9}$ , and  $\frac{6}{12}$ .
- b. **Fractional Parts:**  $\frac{1}{3}$  of 100
- c. **Money:** The price of the used car is \$5000. To buy the car, Sanjay had to make a down payment (first payment) of 10% of the price. What is 10% of \$5000?
- d. **Money:** Sanjay decided to make a greater down payment than was required. He made a down payment of  $\frac{1}{5}$  of \$5000. What is  $\frac{1}{5}$  of \$5000?
- e. **Powers/Roots:**  $2^3$
- f. **Probability:** The bag contains five tiles. Each tile had a vowel written on it. If Stuart reaches into the bag and pulls out one tile without looking, what is the probability it will be the letter C?
- g. **Calculation:**  $\sqrt{100}$ ,  $\times 2$ ,  $\times 50$ ,  $- 1$ ,  $\div 9$
- h. **Roman Numerals:** Write 13 in Roman numerals.

Choose an appropriate problem-solving strategy to solve this problem. Recall that a *permutation* is an ordered arrangement of objects. Adam, Bianca, and Cantara stood side by side to have their picture taken (A, B, C). Then Bianca and Cantara switched places (A, C, B). List the remaining possible side-by-side arrangements.



## New Concept

In this lesson we will begin dividing by two-digit numbers. Dividing by two-digit numbers is necessary to solve problems like the following:

*One hundred forty-four players signed up for soccer. If the players are separated into 12 equal teams, how many players will be on each team?*

When we divide by a two-digit number, we continue to follow the four steps of division: divide, multiply, subtract, and bring down. When we divide by two-digit numbers, the “divide” step takes a little more thought because we have not memorized the two-digit multiplication facts.

### Example 1

#### Thinking Skill

##### Verify

Why do we write the digit 1 in the quotient above the 5?

#### Divide: $150 \div 12$

We begin by breaking the division into a smaller division problem. Starting from the first digit in 150, we try to find a number that 12 will divide into at least once. Our first smaller division is  $12 \overline{)15}$ . We see that there is one 12 in 15, so we write “1” above the digit 5 of the number 15. Then we multiply, subtract, and bring down.

$$\begin{array}{r} 1 \\ 12 \overline{)150} \\ \underline{12} \phantom{0} \\ 30 \end{array}$$

Now we begin a new division. This time we find  $12 \overline{)30}$ . If we are not sure of the answer, we may need to try more than once to find the number of 12s in 30. We find that there are two 12s in 30. We write “2” above the 0 of 150. Then we multiply and subtract. Since there is no digit to bring down, we are finished. The answer is **12 R 6**.

$$\begin{array}{r} 12 \text{ R } 6 \\ 12 \overline{)150} \\ \underline{12} \phantom{0} \\ \phantom{12} 30 \\ \underline{24} \phantom{0} \\ \phantom{12} 6 \end{array}$$

#### Thinking Skill

##### Connect

Why can we use multiplication to check division?

To check our answer, we multiply 12 by 12 and then add the remainder, which is 6.

$$\begin{array}{r} 12 \\ \times 12 \\ \hline 144 \\ + 6 \text{ remainder} \\ \hline 150 \text{ (check)} \end{array}$$

There are some “tricks” we can use to make dividing by two-digit numbers easier. One trick is to think of dividing by only the first digit.

### Example 2

**Divide:**  $32 \overline{)987}$

We begin by breaking the division into the smaller division problem  $32 \overline{)98}$ . Instead of thinking, “How many 32s are in 98?” we can use the first-digit trick and think, “How many 3s are in 9?” We see “ $32 \overline{)98}$ ” but we think “ $3 \overline{)9}$ .” We try 3 as an answer. Since we are really finding  $32 \overline{)98}$ , we write the 3 above the 8 of 98. Then we multiply 3 by 32, subtract, and bring down.

$$\begin{array}{r} 30 \text{ R } 27 \\ 32 \overline{)987} \\ \underline{96} \phantom{0} \\ 27 \\ \underline{0} \\ 27 \end{array}$$

Now we begin the new division  $32 \overline{)27}$ . Since there is not even one 32 in 27, we write “0” in the answer; then we multiply and subtract. There are no digits to bring down, so we are finished. The answer is **30 R 27**. We can check our answer by multiplying 30 by 32 and then adding the remainder, 27.

$$\begin{array}{r} 32 \\ \times 30 \\ \hline 960 \\ + 27 \text{ remainder} \\ \hline 987 \text{ (check)} \end{array}$$

### Example 3

**Loma Vista School expects an enrollment of 868 students. The principal wants to have about 24 students and one teacher per classroom. About how many teachers are needed for the students at Loma Vista School?**

We will use compatible numbers to estimate the number of teachers needed. We could round 24 down to 20, but 24 is closer to 25, so we choose 25. Now we round 868 to a number compatible with 25. Since we rounded 24 up to 25, we round 868 up to 875. We think of 875 as  $800 + 75$ . Every 100 is four 25s, so  $800 \div 25$  is 32. Since 75 is three 25s, we find that  $875 \div 25 = 35$ . Loma Vista School needs about **35 teachers**.

### Lesson

Use compatible numbers to estimate the quotient in problems **a** and **b**.

**a.**  $11 \overline{)253}$

**b.**  $21 \overline{)253}$

Divide:

**c.**  $31 \overline{)403}$

**d.**  $12 \overline{)253}$

**e.**  $12 \overline{)300}$

$23 \overline{)510}$

One hundred forty-four players signed up for soccer. If the players are separated into 12 equal teams, how many players will be on each team?

Divide. Use the first-digit trick to help with the “divide” step.

h.  $30\overline{)682}$

i.  $32\overline{)709}$

j.  $43\overline{)880}$

k.  $22\overline{)924}$

l.  $22\overline{)750}$

m.  $21\overline{)126}$

n.  $21\overline{)654}$

o.  $41\overline{)910}$

p.  $21\overline{)1290}$

## Written Practice

*Distributed and Integrated*

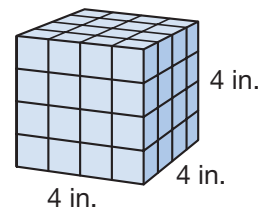
\*1. **Represent** Draw a pair of horizontal line segments. Make them the same length. Then draw two more line segments to make a quadrilateral.  
(31, 32)

2. D’Ron worked on his homework from 3:30 p.m. to 6 p.m. How many minutes did D’Ron work on his homework?  
(28, 49)

3. **Represent** Write a decimal number equal to the mixed number  $3\frac{9}{10}$ .  
(67)

4. If 24 eggs exactly fill 2 cartons, how many eggs will it take to fill 3 cartons?  
(49)

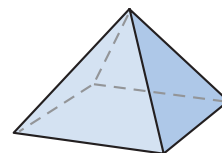
\*5. Some 1-inch cubes were used to build this 4-inch cube.  
(18) How many 1-inch cubes were used?



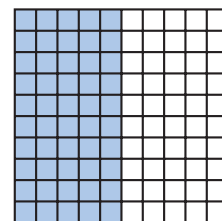
\*6. a. How many apples weighing  $\frac{1}{3}$  pound each would it take to total 1 pound?  
(87)

b. How many apples weighing  $\frac{1}{3}$  pound each would it take to total 4 pounds?

\*7. Name this shape. How many edges does it have?  
(83)

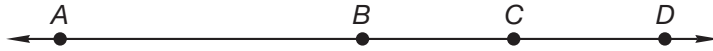


8. Name the shaded portion of this square as a decimal number, as a reduced fraction, and as a percent.  
(71)



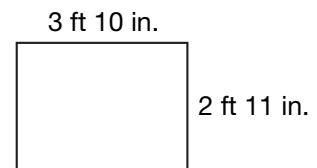
9. **Multiple Choice** Which of these numbers does *not* equal  $\frac{1}{2}$ ?  
 (23) **A** 0.5      **B** 50%      **C**  $\frac{6}{12}$       **D** 0.05

- \* 10.  $AB$  is 40 millimeters.  $BC$  is half of  $AB$ .  $CD$  equals  $BC$ . Find  $AD$ .  
 (61)



11.  $8.7 + 6.25$  (73)      12.  $12.75 - 4.2$  (73)      \* 13.  $4^3$  (78)
14.  $8 \times \$125$  (17)      15.  $\sqrt{100} - \sqrt{64}$  (78)      \* 16.  $293 \div 13$  (92)
- \* 17.  $24 \overline{)510}$  (92)      \* 18.  $3\frac{5}{8} + 1\frac{7}{8}$  (91)      19.  $5 - 1\frac{2}{5}$  (63)
- \* 20.  $\frac{1}{3}$  of 5 (86)      21.  $\frac{3}{4} \times \frac{4}{3}$  (76)      \* 22.  $\frac{6}{10} \div \frac{1}{5}$  (87)
- \* 23. **Analyze** Write a fraction equal to  $\frac{2}{5}$  that has a denominator of 10. Add that fraction to  $\frac{1}{10}$ . Remember to reduce your answer. (79, 81)

- \* 24. **Estimate** The figure shows the length and width of a rectangle. Estimate the area of the rectangle. (62, 72)




25. **Predict** A penny, nickel, dime, and quarter are tossed at the same time. Which word best describes the following events: *likely*, *unlikely*, *certain*, or *impossible*? (57)
- All of the upturned faces are heads.
  - At least one of the upturned faces is heads.
  - There is one more heads than there is tails.
26. **Analyze** In the 1988 Summer Olympic games in Seoul, South Korea, U.S. athlete Florence Griffith-Joyner won three gold medals in track events. "Flo-Jo," as she was called, finished the 200-meter run in 21.34 seconds, breaking the previous Olympic record of 21.81 seconds. By how much did Florence Griffith-Joyner break the previous Olympic record? (73)

27. Use the information below to answer parts **a–c**.  
*(Inv. 5, 62)*


*Sumi, Lupe, and Melanie bought decorations for the party. The table shows the items they purchased.*

Napkins . . . .	\$2.19
Plates . . . . .	\$1.19
Balloons . . . .	\$3.87
Streamers . .	\$1.39

- a.  **Estimate** Describe how to estimate the total cost of the items. What is your estimate?
- b. What was the total cost of the decorations?
- c. If the girls share the cost evenly, how much will each girl pay?

- \*28. **Conclude** If the sequence below repeats after 5 terms, what are the next 5 terms?  
*(Inv. 4)*

4, 4, 1, 4, 4, ...

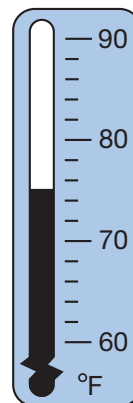
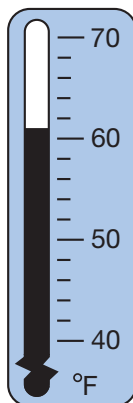
- \*29.  **Represent** The lengths of several suspension bridges in North America are shown in this table:  
*(Inv. 7)*

**Suspension Bridges  
(North America)**

Bridge	Location	Length (ft)
Tacoma Narrows	Tacoma, WA	2800
Golden Gate	San Francisco Bay, CA	4200
A. Murray Mackay	Halifax, Nova Scotia	1400

Name an appropriate type of graph for the data. Explain your choice, and then graph the data.

30. These thermometers show the average daily minimum and maximum temperatures in Auckland, New Zealand, during the month of January. When compared to the lower temperature, how many degrees warmer is the higher temperature?  
*(27)*



## • Comparative Graphs

### Power Up

#### facts

Power Up I

#### mental math

- Number Sense:** Reduce the fractions  $\frac{5}{20}$ ,  $\frac{5}{15}$ , and  $\frac{5}{10}$ .
- Powers/Roots:**  $3^3$
- Money:** The total fee for 4 children to attend the summer camp was \$436. What was the cost per child?  
(Think:  $\$436 \div 4$ .)
- Percent:** What is 50% of \$100? ... 50% of \$10?  
... 50% of \$1?
- Time:** How many years are in a millennium? How many years are in half of a millennium?
- Estimation:** At the game, 329 fans wore red and 273 fans wore orange. There were 947 fans altogether. Use compatible numbers to estimate how many fans did not wear red or orange.
- Calculation:**  $\frac{1}{3}$  of 6,  $\times 2$ ,  $+ 1$ ,  $\times 5$ ,  $- 1$ ,  $\div 6$
- Roman Numerals:** Write IX in our number system.

#### problem solving

Choose an appropriate problem-solving strategy to solve this problem.

Bob erased some of the digits in a

multiplication problem. He then gave it

to Paolo as a problem-solving exercise. He told Paolo that there are two different possible solutions. Copy Bob's multiplication problem, and find both solutions for Paolo.

$$\begin{array}{r} 2\_ \\ \times \_ \\ \hline 2\_2 \end{array}$$

## New Concept

**Comparative graphs** can be used to display two or more sets of related data.

### Example 1

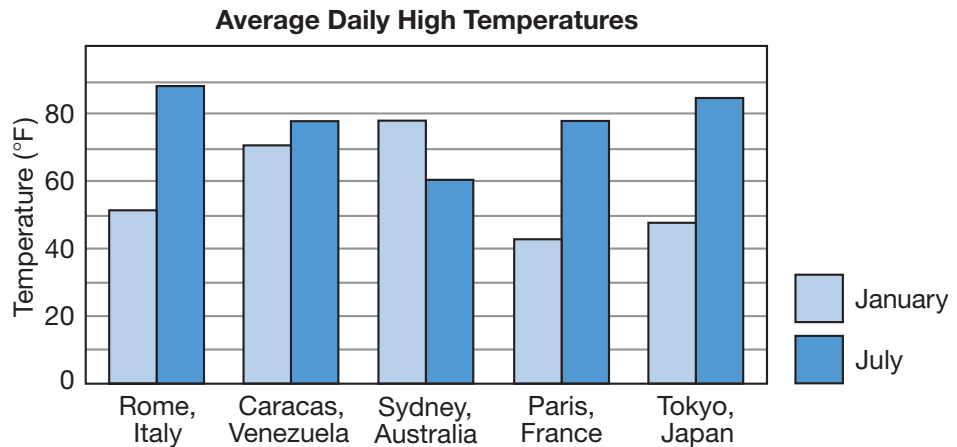
The average daily high temperatures in January and July for five cities is displayed in the comparative vertical bar graph below.

#### Thinking Skill

##### Analyze

How many bars can a bar graph have?

Bar graphs usually do not have a great number of bars. Why not?



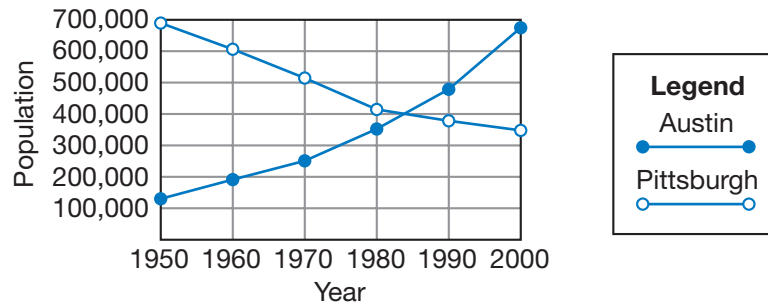
- In which city was the average July high temperature highest?
  - In which city was the average January high temperature lowest?
  - Which city had the smallest range between these temperatures? Do you know why?
  - For which city is the average January high temperature greater than the average July high temperature? Do you know why?
- The tallest dark blue bar appears above **Rome, Italy**. The average July high temperature is about  $89^{\circ}\text{F}$  in Rome.
  - The shortest light blue bar appears above **Paris, France**. The average January high temperature is about  $42^{\circ}\text{F}$  in Paris.
  - The smallest difference in heights of the bars occurs above **Caracas, Venezuela**. Caracas is near the equator, and temperatures in locations near the equator do not vary much throughout the year.
  - We look for the city that has a light blue bar that is taller than its dark blue bar. We find **Sydney, Australia**. Australia is warmer in January than in July because it is south of the equator. South of the equator, January is in the summer and July is in the winter.



Visit [www.SaxonMath.com/Int5Activities](http://www.SaxonMath.com/Int5Activities) for an online activity.

## Example 2

We can use a double-line graph to show how two or more things change in relation to one another. For example, the double-line graph below shows the change in population of the cities of Austin and Pittsburgh from 1950 to 2000. The legend to the right tells which line belongs to which city.



- Approximately what was Austin's population in 1970?
  - Approximately how much did Pittsburgh's population decrease between 1950 and 2000?
- The line graph with solid dots represents Austin's population. For 1970, the dot is about halfway between 200,000 and 300,000, which means the population was **about 250,000**.
  - In the 50-year period, Pittsburgh's population declined from about 700,000 to about 350,000. Subtracting, we find that the decrease was **approximately 350,000**.

$$700,000 - 350,000 = 350,000$$

## Lesson Practice

- Chinara, Alice, Terrell, and Manuel each wrote two stories. The number of paragraphs per story is shown in the table below:

Student	Story 1	Story 2
Chinara	8	8
Alice	3	6
Terrell	6	7
Manuel	7	10

Make a comparative **horizontal** bar graph to show the scores. There should be two bars for each student.

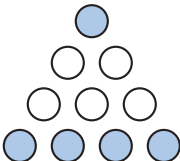


- b. For a science project, Mia and Lonnie each planted a seed. A record of the height of each seedling is shown below. Display the data in a double-line graph.

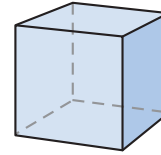
	Week 1	Week 2	Week 3	Week 4
Mia's Seedling	1 cm	5 cm	11 cm	20 cm
Lonnie's Seedling	2 cm	4 cm	10 cm	16 cm

## Written Practice

*Distributed and Integrated*

- \* 1. **Estimate** <sup>(77, 85)</sup> The saying “A pint’s a pound the world around” means that a pint of water weighs about a pound. About how much does 2 quarts of water weigh?
2. <sup>(49)</sup> At a grocery store, apples are sold by the pound. What is the cost of 4 pounds of apples if 3 pounds costs \$2.55?
3. <sup>(46)</sup> If 300 marbles will fill a carton, how many marbles will make the carton  $\frac{1}{2}$  full?
4. <sup>(71)</sup> Name the shaded portion of this group as a decimal number, as a reduced fraction, and as a percent.
- 
- \* 5. a. **Analyze** <sup>(87)</sup> How many plums weighing  $\frac{1}{5}$  pound each would it take to total 1 pound?
- b. How many plums weighing  $\frac{1}{5}$  pound each would it take to total 3 pounds?
6. **Represent** <sup>(8)</sup> Write the following sentence using digits and symbols:  
*When nine is subtracted from twelve, the difference is three.*
- \* 7. <sup>(86)</sup> Compare:  $\frac{2}{3}$  of 3  $\bigcirc$   $3 \times \frac{2}{3}$
8. **Multiple Choice** <sup>(49)</sup> If  $3n = 18$ , then  $2n + 5$  equals which of the following?  
**A** 23                      **B** 17                      **C** 31                      **D** 14

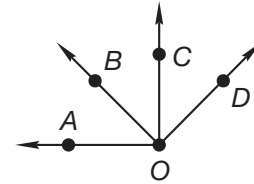
- \*9. A cube has 12 edges. How many edges does a hexagonal prism have?  
(89)



- \*10. **Multiple Choice** Which of these angles appears to be a right angle?  
(31, 61)

- A  $\angle AOB$   
C  $\angle COD$

- B  $\angle BOC$   
D  $\angle AOC$



\*11.  $1\frac{3}{5}$   
(91)  
 $+ 2\frac{4}{5}$   
\_\_\_\_\_

\*12.  $4\frac{5}{8}$   
(90)  
 $- \frac{1}{8}$   
\_\_\_\_\_

\*13.  $6\frac{5}{6}$   
(41)  
 $- 1\frac{5}{6}$   
\_\_\_\_\_

\*14.  $1 \div \frac{1}{8}$   
(87)

\*15.  $\frac{8}{10} \times \frac{5}{10}$   
(90)

\*16.  $\frac{1}{5} \div \frac{1}{10}$   
(87)

\*17.  $12.34 - (5.67 - 0.8)$   
(24, 73)

\*18.  $(\$20 - \$6.55) \div 5$   
(13, 26)

\*19.  $10 \times 56\text{¢}$   
(70)

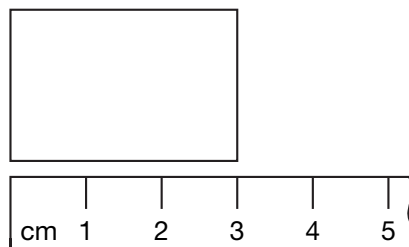
\*20.  $6 \times 78 \times 900$   
(18, 29)

\*21.  $31 \overline{)970}$   
(92)

\*22.  $9^2 - \sqrt{9}$   
(78)

- \*23. **Analyze** Write fractions equal to  $\frac{3}{4}$  and  $\frac{1}{6}$  that have denominators of 12. Then add the fractions.  
(79)

- \*24. Look at the picture below. Then answer parts a-c.  
(53, 72)



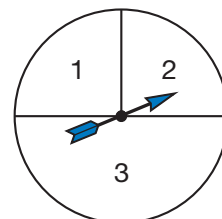
- How long is the rectangle?
- The rectangle is 1 centimeter longer than it is wide. What is the perimeter of the rectangle?
- What is the area of the rectangle?


- \*25.** a. Write the next three terms of the repeating sequence below.  
(Inv. 4)
- b. What is the period of the sequence?
- c. What transformation is shown in the sequence?

**E, W, Ǝ, M, E, \_\_\_\_\_, \_\_\_\_\_, \_\_\_\_\_, ...**

Refer to the spinner to answer parts **a–c**.

- 26.** a. If you spin this spinner 60 times, about how many times  
(Inv. 9) would you expect it to stop on 2?
- b. What percent of the spinner's face is region 2?
- c. What decimal part of the spinner's face is region 3?



- 27.** Montana became a state in 1889, which was 98 years after Vermont  
(49) became a state. Utah became a state 105 years after Vermont. In what year did Utah become a state?
- 28.**  **Estimate** A physical education teacher must divide a class of  
(21, 62) 31 students into four teams. If possible, the same number of students are to be on each team. What is a reasonable estimate of the number of students that will be on each team? Explain your answer.
- 29.** The famous Austrian composer Wolfgang Amadeus Mozart was born  
(35) in 1756. About how many years ago was he born? Explain why your estimate is reasonable.

- \*30.** A square field that is one hectare is 10,000 square meters. Describe  
(78) how to use a calculator to find the length of each side of the field. How long is each side?



## Early Finishers

Real-World Connection

Bryce surveyed students at his school to see if they enjoyed certain activities. The chart below shows the results of the survey. Display the data in a double-line graph. Be sure to label your graph appropriately.

Activity	Boys	Girls
Swimming	55	60
Biking	20	25
Baseball	40	25
Camping	45	35

## • Using Estimation When Dividing by Two-Digit Numbers

### Power Up

#### facts

Power Up I

#### mental math

- Number Sense:** Reduce the fractions  $\frac{3}{15}$ ,  $\frac{5}{15}$ , and  $\frac{10}{15}$ .
- Fractional Parts:**  $\frac{1}{3}$  of 15
- Fractional Parts:**  $\frac{2}{3}$  of 15
- Percent:** 50% of 15
- Geometry:** A soccer ball represents which geometric solid?
- Estimation:** Choose the more reasonable estimate for the mass of a soccer ball: 15 oz or 15 kg.
- Calculation:**  $\sqrt{81}$ ,  $\times 5$ ,  $- 1$ ,  $\div 4$ ,  $+ 1$ ,  $\div 4$ ,  $- 3$
- Roman Numerals:** Write 20 in Roman numerals.

#### problem solving

Choose an appropriate problem-solving strategy to solve this problem. Two cups equal a pint, and two pints equal a quart. Two quarts equal a half gallon. Two half gallons equal one gallon. A quart of milk was poured out of a full gallon container. How many pints of milk were still in the container?

### New Concept

In Lesson 92, we learned a method to help us divide by two-digit numbers. The problems in that lesson were chosen so that using the first digit to guess the division answer would work. However, this method does not always work. In this lesson we will learn another strategy for two-digit division.

Using the first-digit trick for  $19\overline{)59}$ , we follow this process:

We see:      We think:      We try the guess, but  
the guess is too large:

$$19\overline{)59} \xrightarrow{?} 1\overline{)5} \xrightarrow{\textcircled{5}} 19\overline{)59} \begin{array}{r} 5 \\ \underline{95} \end{array}$$

### Reading Math

We know our guess is too large when the number we are subtracting is greater than the number we are subtracting from.

Our guess, 5, is incorrect because there are not five 19s in 59. Our guess is too large. So we will **estimate**. To estimate, we mentally round both numbers to the nearest 10. Then we use the first-digit trick with the rounded numbers.

We see:      We round:      We think:      We try:

$$19\overline{)59} \xrightarrow{} 20\overline{)60} \xrightarrow{} 2\overline{)6} \xrightarrow{\textcircled{3}} 19\overline{)59} \begin{array}{r} 3 \text{ R } 2 \\ \underline{57} \\ 2 \end{array}$$

### Example

**Divide:  $19\overline{)595}$**

We begin by breaking the division into the smaller division problem  $19\overline{)59}$ . We round to  $20\overline{)60}$  and focus on the first digits,  $2\overline{)6}$ . We guess 3, so we write the “3” above the 9 of 59. Then we multiply 3 by 19, subtract, and bring down. The next division is  $19\overline{)25}$ . We may estimate to help us divide. We write “1” in the answer; then we multiply and subtract.

$$\begin{array}{r} 31 \text{ R } 6 \\ 19\overline{)595} \\ \underline{57} \\ 25 \\ \underline{19} \\ 6 \end{array}$$

The answer is **31 R 6**. To check our answer, we multiply 31 by 19 and add the remainder, which is 6.

### Thinking Skill

#### Verify

Why do we write the digit 3 in the tens place of the quotient?

### Lesson Practice

Divide:

a.  $19\overline{)792}$

b.  $30\overline{)600}$

c.  $29\overline{)121}$

d.  $29\overline{)900}$

e.  $48\overline{)829}$

f.  $29\overline{)1210}$

g.  $28\overline{)896}$

h.  $18\overline{)782}$

i.  $39\overline{)1200}$

# Written Practice

Distributed and Integrated

\*1. **List** Write all of the prime numbers less than 50 that end with the digit 1.  
(80)

2. What number is missing in this division problem?  
(20)

$$\square \div 8 = 24$$

3. Sofia ran 660 yards in 3 minutes. At this rate, how many yards would she run in 6 minutes?  
(49)

4. **Represent** Write a decimal number equal to the mixed number  $4\frac{9}{10}$ .  
(71)

5. Seventy-six trombone players led the parade. If they marched in 4 equal rows, how many were in each row?  
(21)

6. a. A dime is what fraction of a dollar?  
(87)

b. How many dimes are in \$1?

c. How many dimes are in \$4?

\*7. **Multiple Choice** Which of the following means, "How many 19s are in 786?"  
(92)

A  $19 \div 786$

B  $786 \div 19$

C  $19 \times 786$

D  $786 \times 19$

\*8. a. How many  $\frac{1}{4}$ s are in 1?  
(87)

b. How many  $\frac{1}{3}$ s are in 1?

9. What word names this shape?  
(83)



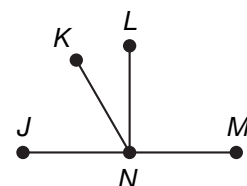
\*10. **Multiple Choice** If  $\overline{LN}$  is perpendicular to  $\overline{JM}$ , then  $\angle JNL$  is what type of angle?  
(31, 61)

A acute

B right

C obtuse

D straight



11.  $\$63.75 + \$1.48 + 59\text{¢} + \$5$   
(70)

12.  $1010 - (101 - 10)$   
(24)

13.  $\$3.48 \times 7$   
(17)

14.  $25^2$   
(78)

\* 15.  $19 \overline{)786}$   
(94)

\* 16.  $\sqrt{36} + \sqrt{64}$   
(78)

\* 17.  $38 \overline{)1200}$   
(94)

\* 18.  $\frac{5}{6} + \frac{5}{6} + \frac{5}{6}$   
(91)

\* 19.  $\frac{5}{6} \times 3$   
(86, 91)

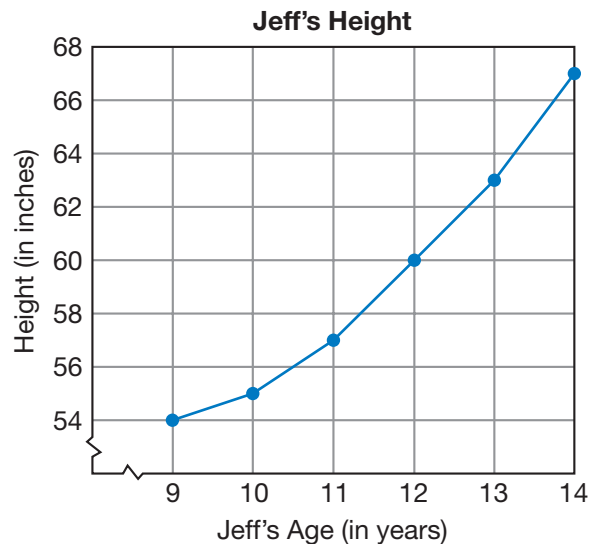
\* 20. Reduce:  $\frac{8}{12}$   
(90)

21.  $3 - \left(2 - \frac{1}{4}\right)$   
(24, 63)

22.  $\frac{1}{3}$  of  $\frac{3}{4}$   
(76)

23. **Analyze** Write a fraction equal to  $\frac{2}{3}$  that has a denominator of 12. Subtract that fraction from  $\frac{11}{12}$ . Remember to reduce the answer.  
(79, 81)

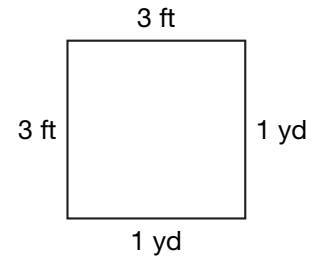
\* 24. **Interpret** The graph below shows Jeff's height from ages 9 to 14. Use this graph to answer parts **a** and **b**.  
(Inv. 6, Inv. 8)



a. How many inches did Jeff grow between the ages of 12 and 14?


b. At what age was Jeff 5 feet tall?


- \*25. The sides of this square are one yard long. Since 1 yard equals 3 feet, the sides are also 3 feet long. Refer to this figure to answer parts a–c.  
(45, 72)



- a. **Multiple Choice** Which of these terms does *not* describe the figure?
- A** rectangle                      **B** parallelogram  
**C** pentagon                      **D** regular quadrilateral
- b. What is the perimeter of the square in feet? What is the perimeter in yards?
- c. What is the area of the square in square feet? What is the area in square yards?

26. a. Compare: 1 yd ○ 3 ft  
(72, 74)  
b. Compare: 1 sq. yd ○ 9 sq. ft

27.  **Explain** In a fifth grade class,  $\frac{7}{8}$  of the students wore sneakers on Friday. What fraction of the class did not wear sneakers on Friday? Explain why your answer is reasonable.  
(43)

28.  **Estimate** In four different rooms of a museum, a curator is planning to display a collection of 152 archeological objects. Estimate the number of objects in each room if each room is to contain the same number of objects. Explain why your estimate is reasonable.  
(62)

29. When an opossum is active, its body temperature is about 95°F. When it is hibernating, its body temperature decreases by about 44°F. What is the body temperature of a hibernating opossum?  
(27)

- \*30. Shakir and Jasmine were both born in 1997. Shakir was born on October 29. Jasmine was born on December 1. How many days after the birth of Shakir was Jasmine born?  
(35)



## • Reciprocals

### Power Up

#### facts

#### Power Up I

#### mental math

- Number Sense:** What is the reduced mixed number for  $\frac{10}{4}$ ?
- Number Sense:** What is the reduced mixed number for  $\frac{10}{6}$ ?
- Number Sense:** What is the reduced mixed number for  $\frac{10}{8}$ ?
- Fractional Parts:**  $\frac{1}{5}$  of 15
- Fractional Parts:**  $\frac{2}{5}$  of 15
- Fractional Parts:**  $\frac{3}{5}$  of 15
- Calculation:**  $9^2, + 9, \div 10, + 9, \div 9$
- Roman Numerals:** Write XXVI in our number system.

#### problem solving

Choose an appropriate problem-solving strategy to solve this problem. All squares are similar. Each side of this square is  $\frac{1}{2}$  inch long. Draw a square with sides half as long and another square with sides twice as long. Calculate the sum of the perimeters of all three squares.



$\frac{1}{2}$  in.

### New Concept

If we switch the numerator and denominator of a fraction, the new fraction is the **reciprocal** of the first fraction. The reciprocal has the same terms, but their positions are reversed. When we switch the positions of the numerator and denominator, we **invert** the fraction.

The reciprocal of  $\frac{2}{3}$  is  $\frac{3}{2}$ .

The reciprocal of  $\frac{3}{2}$  is  $\frac{2}{3}$ .

### Thinking Skill

#### Model

Use the fraction manipulatives to show the reciprocal of  $\frac{1}{4}$ .

Whole numbers also have reciprocals. Recall that a whole number may be written as a fraction by writing a 1 under the whole number. So the whole number 2 may be written as  $\frac{2}{1}$ . To find the reciprocal of  $\frac{2}{1}$ , we invert the fraction and get  $\frac{1}{2}$ .

Since  $2 = \frac{2}{1}$ , the reciprocal of 2 is  $\frac{1}{2}$ .

Notice that the product of  $\frac{1}{2}$  and 2 is 1.

$$\frac{1}{2} \times 2 = 1$$

**The product of any number and its reciprocal is 1.**

$$\frac{2}{3} \times \frac{3}{2} = \frac{6}{6} = 1 \quad \frac{1}{2} \times \frac{2}{1} = \frac{2}{2} = 1$$

Notice that reciprocals appear when we ask these division questions:

How many  $\frac{1}{2}$ s are in 1? Answer: 2 (or  $\frac{2}{1}$ )

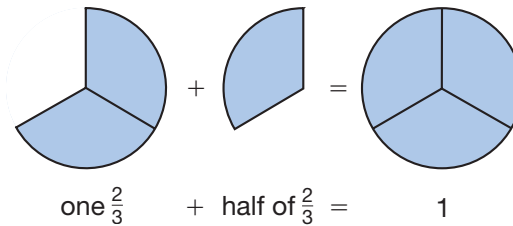
How many  $\frac{1}{3}$ s are in 1? Answer: 3 (or  $\frac{3}{1}$ )

How many  $\frac{1}{4}$ s are in 1? Answer: 4 (or  $\frac{4}{1}$ )

How much of 4 is in 1? Answer:  $\frac{1}{4}$

The reciprocal also appears as the answer to this question:

How many  $\frac{2}{3}$ s are in 1? Answer:  $1\frac{1}{2}$  (or  $\frac{3}{2}$ )



When we divide 1 by any number (except 0), the answer is the reciprocal of the number.

### Example 1

**What is the reciprocal of  $\frac{5}{6}$ ?**

The reciprocal of  $\frac{5}{6}$  is  $\frac{6}{5}$ . We leave the answer as an improper fraction.

**Example 2****What is the product of  $\frac{1}{3}$  and its reciprocal?**The reciprocal of  $\frac{1}{3}$  is  $\frac{3}{1}$ . To find the product, we multiply.

$$\frac{1}{3} \times \frac{3}{1} = 1$$

The product of any fraction and its reciprocal is **1**.**Example 3****What is the reciprocal of 4?**To find the reciprocal of a whole number, we may first write the whole number as a fraction by writing a 1 under it. To write 4 as a fraction, we write  $\frac{4}{1}$ . The reciprocal of  $\frac{4}{1}$  is  $\frac{1}{4}$ .**Example 4****Divide:**  $1 \div \frac{3}{4}$ This problem means, "How many  $\frac{3}{4}$ s are in 1?" When we divide 1 by any number other than zero, the quotient is the reciprocal. So the answer to this division is the reciprocal of  $\frac{3}{4}$ , which is  $\frac{4}{3}$ , or  $1\frac{1}{3}$ . We check the answer by multiplying the quotient  $\frac{4}{3}$  by the divisor  $\frac{3}{4}$ .

$$\frac{4}{3} \times \frac{3}{4} = \frac{12}{12} = 1$$

The result is the original dividend, 1, so the answer is correct.

**Lesson Practice**Write the reciprocal of each number in problems **a–l**. Leave improper fractions as improper fractions.

**a.**  $\frac{4}{5}$

**b.**  $\frac{6}{5}$

**c.** 3

**d.**  $\frac{7}{8}$

**e.**  $\frac{3}{8}$

**f.** 5

**g.**  $\frac{3}{10}$

**h.**  $\frac{5}{12}$

**i.** 2

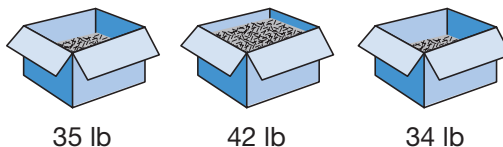
**j.**  $\frac{1}{5}$

**k.** 10

**l.** 1

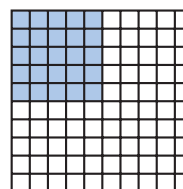
**m.** How many  $\frac{3}{5}$ s are in 1?**n.** Divide:  $1 \div \frac{4}{5}$ **o.** **Analyze** Think of a fraction and write it down. Then write its reciprocal. Multiply the two fractions. What is the product? (Be sure to show your work.)**p.** Is the following sentence true or false?*If the product of two numbers is 1, then the two numbers are reciprocals.*

1. **Analyze** These three boxes of nails weigh <sup>(50)</sup> 35 lb, 42 lb, and 34 lb. If some nails are moved from the heaviest box to the other two boxes so that all three boxes weigh the same, how much will each box weigh?



2. **Analyze** Each finger of the human hand is formed by three bones, <sup>(11)</sup> except for the thumb, which is formed by two bones. The palm contains five bones, one leading to each finger. Not counting the bones in the wrist or thumb, the hand contains how many bones?

3. <sup>(71)</sup> Name the shaded portion of this square as a decimal number, as a reduced fraction, and as a percent.

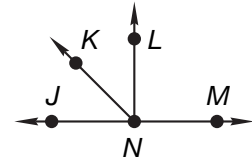


- \*4. <sup>(95)</sup> What is the product of  $\frac{2}{3}$  and its reciprocal?
5. <sup>(87)</sup> a. A quarter is what fraction of a dollar?  
 b. How many quarters equal \$1?  
 c. How many quarters equal \$5?
- \*6. <sup>(95)</sup> What is the reciprocal of  $\frac{3}{4}$ ? What is the product of  $\frac{3}{4}$  and its reciprocal?
- \*7. **Multiple Choice** <sup>(92)</sup> Which of the following means “How many 25s are there in 500?”  
 A  $25 \div 500$       B  $500 \div 25$       C  $25 \times 500$       D  $500 \times 25$
- \*8. <sup>(95)</sup> a. What is the reciprocal of 6?  
 b. What is the reciprocal of  $\frac{1}{4}$ ?

- \*9. Multiple Choice** If  $LN$  is perpendicular to  $\overline{JM}$ , then which of these angles is an acute angle?

**A**  $\angle LNM$   
**C**  $\angle KNL$

**B**  $\angle JNL$   
**D**  $\angle KNM$



**10.**  $(\$20 - \$4.72) \div 8$   
(13, 26)

**11.**  $160 \times \$1.25$   
(56)

**12.**  $25.45 - (1.4 + 0.28)$   
(24, 73)

**13.**  $100^2$   
(78)

**\*14.**  $31 \overline{)140}$   
(94)

**\*15.**  $27x = 567$   
(18)

**\*16.** Reduce:  $\frac{15}{25}$   
(90)

**\*17.**  $1\frac{5}{6} + 1\frac{5}{6}$   
(91)

**18.**  $4\frac{5}{6} - 1\frac{1}{6}$   
(81)

**\*19.**  $\frac{3}{8}$  of 24  
(86)

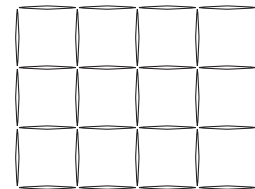
**\*20.**  $3 \times \frac{4}{5}$   
(86, 91)

**\*21.**  $\frac{9}{10} \div \frac{1}{10}$   
(87)

- \*22. Analyze** Write fractions equal to  $\frac{3}{4}$  and  $\frac{1}{6}$  that have denominators of 12. Subtract the smaller fraction from the larger fraction.

- \*23.** The giant pendulum swung back and forth 10 times in 123 seconds. The pendulum swung back and forth one time in how many seconds?

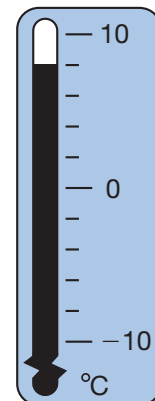
- 24. Connect** Isabella used toothpicks to make this rectangle. Refer to this rectangle to answer parts **a** and **b**.



**a.** How many toothpicks form the perimeter of this rectangle?

**b.** The rectangle encloses an area covered with small squares. How many small squares cover the area of the rectangle?

- 25.** Nicholas awoke on a cool fall morning and looked at the thermometer outside his window. What temperature is indicated on the thermometer?



26. Copy the thermometer from problem 25 on your paper.  
 (27) Then draw another thermometer, and write the Fahrenheit temperatures for  $10^{\circ}\text{C}$ ,  $0^{\circ}\text{C}$ , and  $-10^{\circ}\text{C}$ . (Recall that a difference of  $10^{\circ}\text{C}$  is equal to a difference of  $18^{\circ}\text{F}$ .)


27.  $\sqrt{100} - \sqrt{36}$   
 (78)

28. **Multiple Choice** Ayanna tossed two standard number cubes. She needs to roll a 12 to win the game. What word best describes her chances of rolling 12 in one try?

- A certain      B likely      C unlikely      D impossible

\*29. a. **Multiple Choice** Which of these Venn diagrams illustrates the relationship between squares (S) and rhombuses (R)?  
 (Inv. 7)

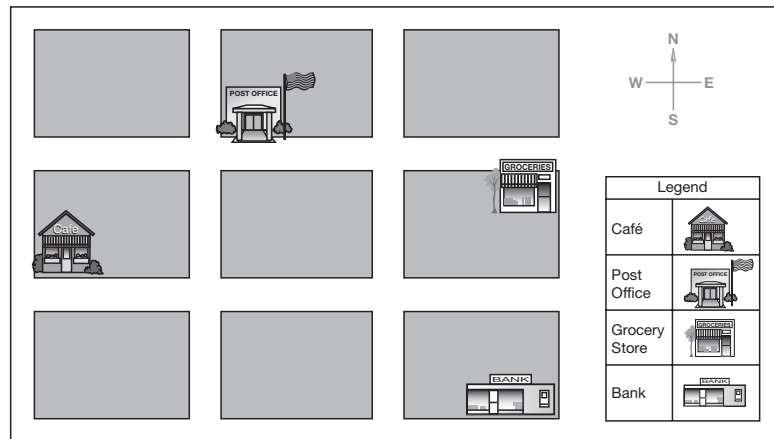


b.  **Explain** Explain your answer for part a.

\*30. The area of Fort Worth is 139.1 square miles greater than the area of Denver. The area of Denver is 67.7 square miles greater than the area of Honolulu. The area of Fort Worth is 292.5 square miles. What is the area of Honolulu?

**Early Finishers**  
 Real-World Connection

The map below shows the location of several places in town where the mail truck stops. Refer to the map to complete parts a and b.



- a. How many blocks would the mail truck travel if it took the shortest path to the bank from the café?
- b. Compare the shortest distance between the post office and the bank to the shortest distance between the grocery store and the café.

## • Using Reciprocals to Divide Fractions

### Power Up

#### facts

Power Up I

#### mental math

- Number Sense:** What is the reduced mixed number for  $\frac{14}{4}$ ?
- Number Sense:** What is the reciprocal of  $\frac{5}{6}$ ?
- Fractional Parts:** Tamara cooked  $\frac{1}{4}$  of the dozen eggs for breakfast. How many eggs did she cook?
- Fractional Parts:** In many parts of the country, school is in session for approximately  $\frac{3}{4}$  of a year. How many months is  $\frac{3}{4}$  of a year?
- Powers/Roots:**  $2^2 + 3^2$
- Geometry:** A soup can represents which geometric solid?
- Time:** Kelly boarded the school bus at 7:34 a.m. The bus arrived at school 23 minutes later. At what time did Kelly arrive at school?
- Roman Numerals:** Write 34 in Roman numerals.

#### problem solving

Choose an appropriate problem-solving strategy to solve this problem. Kerry is wearing a necklace with 30 beads strung in a red-white-blue-red-white-blue pattern. If she counts beads in the direction shown, starting with red, what will be the color of the one hundredth bead?



### New Concept

Reciprocals can help us solve division problems such as the following:

$$\frac{1}{2} \div \frac{2}{3}$$

**Math Language**

Two numbers whose product is 1 are called *reciprocals*.

This problem means “How many  $\frac{2}{3}$ s are in  $\frac{1}{2}$ ?” However, since  $\frac{2}{3}$  is more than  $\frac{1}{2}$ , the answer is less than 1. So we change the question to

“How much of  $\frac{2}{3}$  is in  $\frac{1}{2}$ ?”

“How much of  is in  ?”

This problem is different from the problems we have been solving. To solve this problem, we will use another method. This method uses reciprocals to help us find the answer. We begin by asking a different question: “How many  $\frac{2}{3}$ s are in 1?” Once we know how many  $\frac{2}{3}$ s are in 1, then we can find how much of  $\frac{2}{3}$  is in  $\frac{1}{2}$ .

**Step 1:** How many  $\frac{2}{3}$ s are in 1? The answer is  $\frac{3}{2}$ , which is the reciprocal of  $\frac{2}{3}$ .

**Step 2:** The number of  $\frac{2}{3}$ s in  $\frac{1}{2}$  is *half* the number of  $\frac{2}{3}$ s in 1. So we multiply  $\frac{3}{2}$  by  $\frac{1}{2}$ .

$$\frac{1}{2} \times \frac{3}{2} = \frac{3}{4}$$

This method changes the division problem into a multiplication problem. Instead of dividing  $\frac{1}{2}$  by  $\frac{2}{3}$ , we end up multiplying  $\frac{1}{2}$  by the reciprocal of  $\frac{2}{3}$ .

$$\begin{array}{r} \frac{1}{2} \div \frac{2}{3} = ? \\ \frac{1}{2} \downarrow \frac{3}{2} \\ \frac{1}{2} \times \frac{3}{2} = \frac{3}{4} \end{array}$$

**Discuss** The answer  $\frac{3}{4}$  represents part of what whole? Explain.

**Example 1**

**Divide:**  $\frac{2}{3} \div \frac{1}{2}$

We are finding the number of  $\frac{1}{2}$ s in  $\frac{2}{3}$ . The number of  $\frac{1}{2}$ s in 1 is  $\frac{2}{1}$ . So the number of  $\frac{1}{2}$ s in  $\frac{2}{3}$  is  $\frac{2}{3}$  of  $\frac{2}{1}$ . We multiply  $\frac{2}{3}$  by the reciprocal of the second fraction,  $\frac{1}{2}$ . We simplify the answer  $\frac{4}{3}$  to get  $1\frac{1}{3}$ .

$$\begin{array}{r} \frac{2}{3} \div \frac{1}{2} \\ \frac{2}{3} \downarrow \frac{2}{2} \\ \frac{2}{3} \times \frac{2}{1} = \frac{4}{3} \\ = 1\frac{1}{3} \end{array}$$



## Example 2

**Divide:**  $2 \div \frac{2}{3}$

We are finding the number of  $\frac{2}{3}$ s in 2. The number of  $\frac{2}{3}$ s in 1 is  $\frac{3}{2}$ . So the number of  $\frac{2}{3}$ s in 2 is twice that many. We write the whole number 2 as the fraction  $\frac{2}{1}$ . Then we multiply  $\frac{2}{1}$  by the reciprocal of  $\frac{2}{3}$ . Finally, we simplify the answer and find that the number of  $\frac{2}{3}$ s in 2 is **3**.

$$\begin{array}{r} \frac{2}{1} \div \frac{2}{3} \\ \frac{2}{1} \downarrow \frac{3}{2} \\ \frac{2}{1} \div \frac{3}{2} = \frac{6}{2} \\ = 3 \end{array}$$

**Justify** Why is the answer reasonable? Use  $1 \div \frac{2}{3}$  in your explanation.

## Lesson Practice

Divide:

a.  $\frac{1}{3} \div \frac{1}{2}$

b.  $\frac{2}{3} \div \frac{3}{4}$

c.  $\frac{2}{3} \div \frac{1}{4}$

d.  $\frac{1}{2} \div \frac{1}{3}$

e.  $\frac{3}{4} \div \frac{2}{3}$

f.  $3 \div \frac{3}{4}$

g.  $2 \div \frac{1}{3}$

h.  $3 \div \frac{2}{3}$

i.  $10 \div \frac{5}{6}$

j. How many  $\frac{1}{3}$ s are in  $\frac{3}{4}$ ?

k. How much of  $\frac{3}{4}$  is in  $\frac{1}{3}$ ?

## Written Practice

*Distributed and Integrated*

- 1. Represent** (37) Draw two circles. Shade  $\frac{1}{2}$  of one circle and  $\frac{2}{3}$  of the other circle.
- 2. Analyze** (71, 76) James gave Ramone half of an apple. Ramone gave his sister half of what he had. What fraction of the whole apple did Ramone's sister get? What percent of the whole apple did she get?
- 3.** (86) How much is  $\frac{2}{3}$  of one dozen?
- 4. Estimate** (62) Write the product of 712 and 490 by rounding both numbers to the nearest hundred before multiplying.
- 5. Represent** (52) Use digits to write the number ninety-three million, eight hundred fourteen thousand, two hundred.

**\*6. Multiple Choice** Which of these means “How many one tenths are there in three?”

(87)

- A**  $\frac{1}{10} \div 3$       **B**  $3 \div \frac{1}{10}$       **C**  $\frac{1}{10} \div \frac{3}{10}$       **D**  $\frac{3}{10} \div \frac{1}{10}$

**\*7. Analyze** Write fractions equal to  $\frac{1}{4}$  and  $\frac{1}{5}$  that have denominators of 20. Then add the fractions.

(79)

**\*8. a.**  $1 \div \frac{1}{10}$       **b.**  $3 \div \frac{1}{10}$

(96)

**9.** Recall that the multiples of a number are the numbers we say when counting by that number. The first four multiples of 2 are 2, 4, 6, and 8. What are the first four multiples of 3?

(15)

**\*10.** The blossom of the saguaro cactus is the state flower of Arizona. A saguaro cactus can weigh as much as 10 tons. About  $\frac{3}{4}$  of a saguaro’s weight comes from the water it stores inside of it. If a saguaro cactus weighs 10 tons, about how much of its weight is water?

(46, 86)

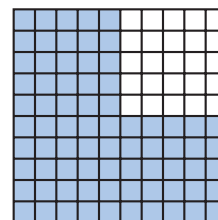
**11.**  $AB$  is 3 cm.  $BC$  is 4 cm.  $AD$  is 10 cm. Find  $CD$ .

(61)



**12. Connect** Name the shaded portion of this square as a decimal number, as a reduced fraction, and as a percent.

(71)



**\*13.**  $\frac{1}{3} \div \frac{1}{4}$

(96)

**\*14.**  $\frac{1}{4} \div \frac{1}{3}$

(96)

**\*15.**  $3 \div \frac{1}{2}$

(96)

**16.**  $m + 1.4 = 3.75$

(73)

**17.**  $m - 1.4 = 3.75$

(73)

**18.**  $\frac{1}{10} \times \square = \frac{10}{100}$

(79)

**19.**  $20 \times 47\text{¢} = \$\underline{\hspace{1cm}}$

(70)

**20.**  $568 \div 15$

(94)

**21.**  $30 \overline{)427}$

(54)

**22.**  $6m = \$30.24$

(26, 34)

**\*23.**  $5 \times \left(\frac{2}{3} \times \frac{1}{2}\right)$

(86, 91)

**\*24.**  $5 - \left(1\frac{1}{4} + 2\right)$

(43, 63)

**25.** Compare:  $\sqrt{100} \bigcirc 5^2$

(78)

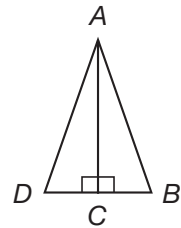
- \*26. **Analyze** (84) At Walton School there are 15 classrooms. The numbers of students in each classroom are listed below. Use this information to answer parts **a–c**.

20, 18, 30, 20, 22, 28, 31, 20, 27, 30, 26, 31, 20, 24, 28

- What is the mode of the number of students in the classrooms?
- What is the range?
- What is the median number of students in the classrooms?

- \*27. (36, 88) In this figure triangles  $ABC$  and  $ADC$  are congruent. Refer to the figure to answer parts **a** and **b**.

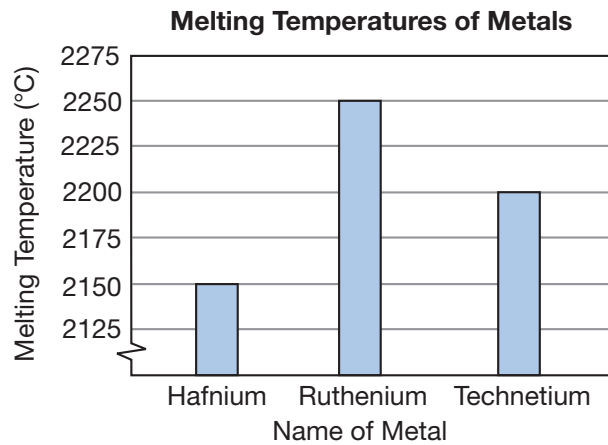
- Classified by sides, what type of triangle is triangle  $ABD$ ?
- What single transformation would move triangle  $ABC$  to the position of triangle  $ADC$ ?




28. (49) Mio's birthday is 6 days before Rusita's birthday and 14 days after Michelle's birthday. Michelle's birthday is July 4. When is Rusita's birthday?

29. (62) **Estimate** Tyrone estimates that his sport utility vehicle travels about 21 miles for every gallon of fuel it uses. Trina estimates that her car will travel about 5 more miles for every gallon of fuel it uses. If each person drives their vehicle for 500 miles, about how many fewer gallons of fuel will Trina need to purchase? Explain why your answer is reasonable.

- \*30. Interpret** (Inv. 7) Gold, iron, and aluminum are well-known examples of metals. The melting temperatures of other metals are shown in the bar graph below. Use the graph to answer the questions that follow.



- What is the range in degrees Celsius of the melting temperatures?
- Which of the three metals melts at the coldest temperature?
-  **Estimate** The melting temperature of gold is  $1064.43^{\circ}\text{C}$ . About how many degrees less than the melting temperature of ruthenium is the melting temperature of gold? Explain why your estimate is reasonable.

**Early Finishers**  
Real-World Connection

Rahul worked through the problem below.

$$\frac{8}{7} \div \frac{9}{6} = \frac{7}{8} \times \frac{6}{9} = \frac{42}{72} = \frac{21}{36} = \frac{7}{12}$$

- What mistake did he make?
- What is the correct answer in simplest form?

## • Ratios

### Power Up

#### facts

#### Power Up I

#### mental math

- Number Sense:** Simplify the improper fractions  $\frac{7}{6}$ ,  $\frac{8}{6}$ , and  $\frac{9}{6}$ .
- Number Sense:** What is the reciprocal of  $\frac{1}{3}$ ?
- Fractional Parts:**  $\frac{1}{3}$  of 100
- Number Sense:**  $33\frac{1}{3} + 33\frac{1}{3}$
- Number Sense:**  $\frac{2}{3}$  of 100
- Powers/Roots:**  $3^2 - 1$
- Calculation:**  $10\%$  of 500,  $\times 10$ ,  $\div 2$ ,  $- 10$ ,  $\div 4$ ,  $+ 3$ ,  $\div 9$
- Roman Numerals:** Write XXXIV in our number system.

#### problem solving

Choose an appropriate problem-solving strategy to solve this problem. George was down to three clean socks, one red, one white, and one blue. How many combinations of two socks can George make from these three socks?

For each combination of two socks, George could choose between two permutations of the socks. For example, George could wear a red sock on his left foot and a white sock on his right foot (R, W), or he could switch the socks (W, R). List all the permutations of two socks George could make.

### New Concept

#### Math Language

A *ratio* is a comparison. In this problem, we are comparing the number of boys in a class to the number of girls.

A **ratio** is a way of comparing numbers by division:

*If there are 12 boys and 18 girls in a class, then the ratio of boys to girls in the class is 12 to 18.*

We often write ratios as fractions. We write the terms of the ratio in order from top to bottom.

The ratio “12 to 18” is written  $\frac{12}{18}$ .

We read the ratio  $\frac{12}{18}$  by saying “twelve to eighteen.”

We reduce ratios just as we reduce fractions. Since 12 and 18 are both divisible by 6, we divide each term of  $\frac{12}{18}$  by 6.

$$\frac{12 \div 6}{18 \div 6} = \frac{2}{3}$$

So the ratio of boys to girls in the class is  $\frac{2}{3}$  (“two to three”). This means that for every two boys in the class, there are three girls.

### Example

**There were 12 girls and 16 boys in the class. What was the ratio of boys to girls?**

First we place the numbers in the correct order. We are asked for the ratio of boys to girls. Since we follow the order from top to bottom, we write the number of boys as the numerator and the number of girls as the denominator.

$$\frac{\text{boys } 16}{\text{girls } 12}$$

Unlike fractions, we do not write ratios as mixed numbers. The top number of a ratio may be greater than the bottom number. However, we do reduce ratios. Since the terms of the ratio, 16 and 12, are both divisible by 4, we reduce the ratio as follows:

$$\frac{16 \div 4}{12 \div 4} = \frac{4}{3}$$

The ratio of boys to girls in the class was  $\frac{4}{3}$ .

**Connect** What is the ratio of girls to boys?

### Thinking Skill

#### Analyze

Extend the diagram below to show the total number of boys and girls in the class.

B B B B  
G G G

### Lesson Practice

There were 20 prairie dogs and 30 jackrabbits in a section of the Texas high plains.

- What was the ratio of jackrabbits to prairie dogs?
- What was the ratio of prairie dogs to jackrabbits?

There were 8 red socks and 10 blue socks in George’s drawer.

- What was the ratio of red socks to blue socks?
- What was the ratio of blue socks to red socks?

\*1. There were 15 pennies and 10 nickels in Kordell's drawer. What was the ratio of pennies to nickels in his drawer?  
(97)

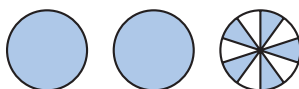
2. **Represent** Write this sentence using digits and symbols:  
(41)

*The sum of one fourth and one fourth is one half.*

3. **Explain** Paige had four \$1 bills, 3 quarters, 2 dimes, and 1 nickel.  
(49) If she spent half of her money, how much money does she have left? Explain how you found your answer.

4. How many  $\frac{1}{8}$ s are in  $\frac{1}{2}$ ?  
(87)

5. **Connect** Name the number of shaded circles as a decimal number and as a reduced mixed number.  
(71, 81)



6. When the decimal number eleven and twelve hundredths is subtracted from twelve and eleven hundredths, what is the difference?  
(68, 73)

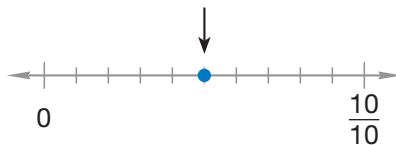
7. a. A quart is what fraction of a gallon?  
(85, 86)

b. How many quarts are in 1 gallon?

c. How many quarts are in 4 gallons?

8. **Analyze** Write fractions equal to  $\frac{2}{3}$  and  $\frac{2}{5}$  that have denominators of 15. Then subtract the smaller fraction from the larger fraction.  
(79)

9. **Connect** Name the point marked by the arrow as a decimal number and as a fraction.  
(66)



\*10. Compare:  $\frac{1}{2} \div 2$   $\bigcirc$   $2 \div \frac{1}{2}$   
(96)

11.  $AB$  is 30 millimeters.  $CD$  is 40 millimeters.  $AD$  is 90 millimeters.  
(61) Find  $BC$ .



\*12.  $3 \div \frac{2}{3}$   
(96)

\*13.  $\frac{2}{3} \div 3$   
(96)

14.  $\frac{7}{10} + \frac{7}{10}$   
(91)

15.  $43.15 + 8.69 + 7.2 + 5.0$   
(73)

16.  $(\$10 - 19\text{¢}) \div 9$   
(24, 70)

17.  $6 \times 72\text{¢} = \$ \underline{\hspace{1cm}}$   
(70)

18.  $35^2$   
(78)

\*19.  $24 \overline{)500}$   
(92)

20. Reduce:  $\frac{50}{100}$   
(90)

\*21.  $12y = 1224$   
(26, 92)

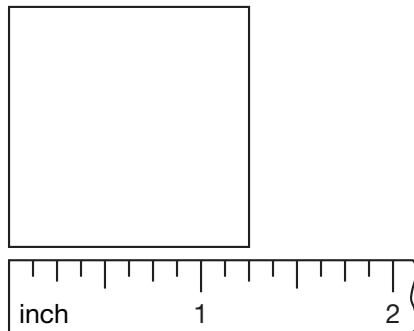
22.  $5\frac{3}{4} - \left(3 - 1\frac{3}{4}\right)$   
(63, 81)

23.  $1\frac{1}{4} + 1\frac{1}{4} + 1\frac{1}{4} + 1\frac{1}{4}$   
(59)

24.  $\frac{3}{10} = \frac{\square}{100}$   
(79)

25. a. What is the length of each side of this square?  
(44, 53)

b. What is the perimeter of this square?



\*26. If the area of a square is 64 square inches, then what is the length of each side?  
(72, 78)

\*27. What number is  $\frac{1}{64}$  of 640?  
(86, 92)

\*28. **Conclude** What are the next three terms in this Fibonacci sequence?  
(Inv. 4)

1, 1, 2, 3, 5, 8, 13, 21, \_\_\_\_\_, \_\_\_\_\_, \_\_\_\_\_, ...



**\*29.** a. List the factors of 64 from least to greatest.  
(78, 84)

b. Is the number of factors an odd or even number?

c. What is the median of the factors?

d. What is  $\sqrt{64}$ ?

**\*30.** There are 50 stars and 13 stripes on the United States flag. What is the ratio of stripes to stars on the flag?  
(97)



**Early  
Finishers**

*Real-World  
Connection*

Mrs. Carter and her family drove to Fond du Lac, Wisconsin. They traveled 200 miles in 4 hours.

a. What is the ratio of miles to hours in lowest terms?

b. Explain the meaning of the ratio you have found.

## • Temperature

### Power Up

#### facts

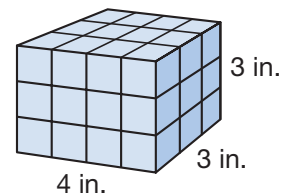
#### mental math

#### Power Up I

- Number Sense:** Simplify the improper fractions  $\frac{6}{4}$ ,  $\frac{7}{4}$ , and  $\frac{8}{4}$ .
- Number Sense:** What is the reciprocal of  $\frac{3}{4}$ ? ... of  $\frac{1}{4}$ ?
- Percent:** What is 50% of \$20? ... 25% of \$20? ... 10% of \$20?
- Powers/Roots:**  $4^2 + 3^2$
- Estimation:** Choose the more reasonable estimate for the capacity of a coffee cup: 300 mL or 300 L.
- Time:** The movie began at 6:45 p.m. It ended 1 hour 50 minutes later. At what time did the movie end?
- Calculation:**  $\frac{1}{3}$  of 21,  $\times 2$ ,  $+ 1$ ,  $\div 3$ ,  $\times 6$ ,  $+ 2$ ,  $\div 4$
- Roman Numerals:** Write 18 in Roman numerals.

#### problem solving

Choose an appropriate problem-solving strategy to solve this problem. Sasha used 1-inch cubes to build this rectangular solid. How many 1-inch cubes did Sasha use?



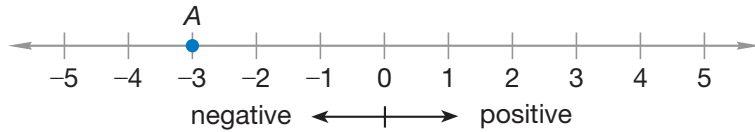
### New Concept

Numbers that are greater than zero are **positive numbers**. Numbers that are less than zero are *negative numbers*. Zero is neither positive nor negative. On the number line on the following page, we show both positive and negative numbers. We write negative numbers with a minus sign in front of the number. Point A is at  $-3$ , which we read as “negative three.”

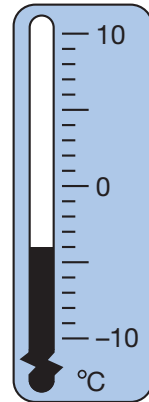
### Thinking Skill

#### Compare

What is the freezing point of water on the Celsius scale? On the Fahrenheit scale?



One place we might see negative numbers is on a thermometer. On a very cold day the temperature may drop below zero. If the temperature is four degrees below zero, we might say the temperature is “minus four.”

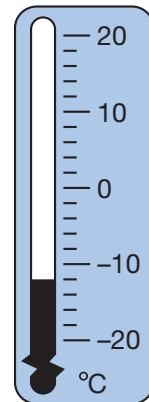


### Example 1

The high temperature for the day was  $6^{\circ}\text{C}$ . The thermometer shows the low temperature during the night. How many degrees are there between the high and low temperatures?

The distance between tick marks is two degrees. Counting down from  $0^{\circ}$ , we find that the thermometer indicates a temperature of  $-12^{\circ}\text{C}$ .

The high temperature was  $6^{\circ}\text{C}$  above zero, so the difference between the high and low temperatures is  $18^{\circ}$ .



### Example 2

The highest temperature recorded in Death Valley is  $134^{\circ}\text{F}$ . The lowest temperature recorded is  $15^{\circ}\text{F}$ . Estimate the difference in the temperatures.

We can use compatible numbers to estimate the difference.

$$135 - 15 = 120$$

There is about a  $120^{\circ}\text{F}$  difference.

### Example 3

Julius recorded the daily high and low temperatures in the desert for a week in the table below.

Daily Temperatures (°C)

Day	Sun	Mon	Tues	Wed	Thurs	Fri	Sat
High (°C)	25	20	32	40	45	30	32
Low (°C)	11	13	15	23	25	15	15

- What was the median high temperature for the week?
- What was the mode of the low temperatures recorded?
- What was the range of temperatures for the week?

a. To find the median high temperature, we first list the high temperatures in numerical order.

20, 25, 30, 32, 32, 40, 45

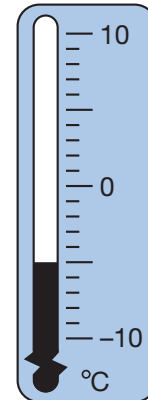
The middle number is 32, so the median is **32°C**.

- The most frequently recorded low temperature is **15°C**.
- The range of temperatures for the week is the difference between the highest high (45) and the lowest low (11). When we subtract, we find that the range is **34°C**.

**Evaluate** Could these temperatures represent a week in your community? Why or why not?

### Lesson Practice


- Use digits and symbols to write the temperature that is twelve degrees below zero on the Fahrenheit scale.
- What temperature is shown on this thermometer?
- If the temperature shown on the thermometer is  $14^\circ$  lower than the high temperature for the day, then what was the high temperature?



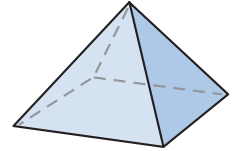
- If the temperature falls  $3^\circ$  from the temperature shown, what will the temperature be?

Refer to the table in Example 3 to answer problems e and f.

- What is the median of the low temperatures for the week?
- What is the mean (average) of the high temperatures for the week?

- \*1.**  **Justify** (97) There were 12 dogs and 8 cats at the class pet show. What was the ratio of cats to dogs at the show? Explain how you found your answer

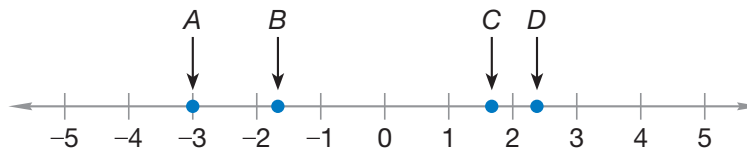
- 2. a.** **Conclude** (83) What is the name of this solid?  
**b.** How many faces does it have?



- 3.** (73) Logan lives 1.2 miles from school. How far does he travel going from home to school and back home?

- 4.** (98) One day in January, the temperature at 9:00 p.m. in Juneau, Alaska, was  $2^{\circ}\text{F}$ . By 11:00 p.m. the temperature had dropped  $5^{\circ}$ . What was the temperature at 11:00 p.m.?

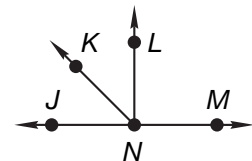
- \*5.** **Connect** (38, 98) Refer to this number line to solve parts **a** and **b**.



- a.** Which arrow could be pointing to  $2\frac{1}{3}$  on the number line?  
**b.** Which arrow is pointing to negative three?
- 6.** (49) A person who is a fast swimmer can swim at a rate of about 5 mph. A trout can swim about 10 mph faster, and a sailfish can swim about 45 mph faster than a trout. About how fast can a sailfish swim?

- \*7. Multiple Choice** (31, 61) If  $\overline{LN}$  is perpendicular to  $\overline{JM}$ , then which of these angles is obtuse?

- A**  $\angle JNK$                       **B**  $\angle KNL$   
**C**  $\angle KNM$                       **D**  $\angle LNM$



8.  $6.5 + 2.47 + 0.875$   
(73)

9.  $4.26 + 8.0 + 15.9$   
(73)

10.  $23.45 - 1.2$   
(73)

11.  $0.367 - 0.1$   
(73)

12.  $\$1.25 \times 7$   
(17)

13.  $750 \times 608$   
(56)

\*14.  $364 \div 16$   
(94)

15.  $\$7.20 \div 20$   
(54)

16.  $3\frac{1}{2}$   
(59)  
 $+ 1\frac{1}{2}$   

---

17.  $5\frac{8}{15}$   
(41)  
 $- 4\frac{7}{15}$   

---

18.  $6$   
(63)  
 $- 1\frac{1}{3}$   

---

19.  $1 \times \frac{5}{7}$   
(86)

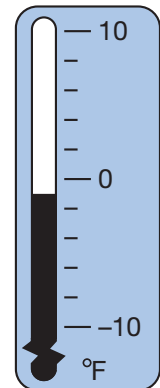
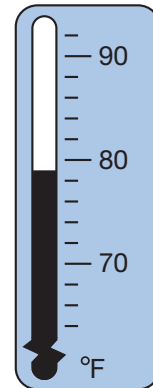
20.  $\frac{4}{5}$  of 25  
(86)

\*21.  $\frac{3}{4} \div \frac{2}{3}$   
(96)

22.  $\frac{7}{10} = \frac{\square}{100}$   
(79)

23. Reduce:  $\frac{30}{100}$   
(90)

- \*24. a. The thermometer at left shows the highest temperature in Madison in December. What is the temperature shown?  
b. The thermometer at right shows the lowest temperature in Madison in December. What is the temperature shown?  
c. What is the range of the two temperatures shown?



25.  $9^2 + \sqrt{81}$   
(78)

- \*26. a. Find the common factors of 70 and 100.  
(82, 90)

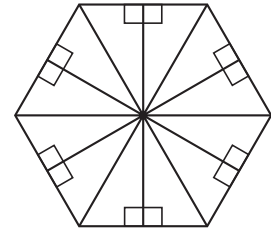
b. Use the GCF of 70 and 100 to reduce  $\frac{70}{100}$ .

27. Compare:  
(76)

a.  $\frac{1}{2} \times \frac{1}{3} \bigcirc \frac{1}{2}$

b.  $\frac{1}{2} \times \frac{1}{3} \bigcirc \frac{1}{3}$

- \*28. Multiple Choice** This regular hexagon contains <sup>(88)</sup> 12 congruent right triangles. Look at one triangle and a second triangle right next to it. Which transformation moves the first triangle to the position of the second triangle?



- \*29. Multiple Choice** Which of these numbers is a composite <sup>(80)</sup> number?

**A** 3                      **B** 5                      **C** 7                      **D** 9

- \*30.** Arrange these decimal numbers in order from least to greatest: <sup>(69)</sup>

0.376   0.037   0.38   0.367

## Early Finishers

Real-World Connection

Maria went scuba diving off the coast of Florida and stayed close to the shore. The deepest she dove was 8 feet below sea level, or  $-8$  feet. Now she is standing on a dune that is 6 feet above sea level, or  $+6$  feet.

- Draw a number line and label the two elevations Maria visited.
- What is the difference between the two points Maria has visited?

## • Adding and Subtracting Whole Numbers and Decimal Numbers

### Power Up

#### facts

Power Up I

#### mental math

- Number Sense:** Simplify the improper fractions  $\frac{10}{8}$ ,  $\frac{11}{8}$ , and  $\frac{12}{8}$ .
- Number Sense:** What is the reciprocal of  $\frac{1}{2}$ ? ... of 2?
- Number Sense:**  $\frac{1}{8}$  of 100
- Number Sense:**  $12\frac{1}{2} + 12\frac{1}{2} + 12\frac{1}{2}$
- Number Sense:**  $\frac{3}{8}$  of 100
- Measurement:** How many feet are in 33 yards? ... in  $33\frac{1}{3}$  yards?
- Calculation:**  $\sqrt{64}$ ,  $\times 6$ ,  $\div 8$ ,  $\times 4$ ,  $\div 3$
- Roman Numerals:** Write XXXII in our number system.

#### problem solving

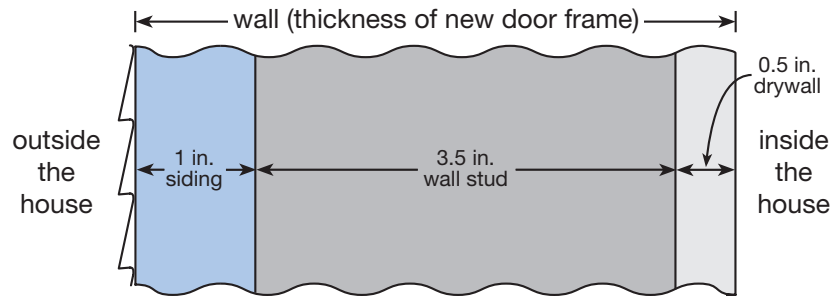
Choose an appropriate problem-solving strategy to solve this problem. Two cups equal a pint. Two pints equal a quart. Two quarts equal a half gallon. Two half gallons equal a gallon. Imani used a gallon container that was full of water to fill a half-gallon container, a quart container, a pint container, and a cup container. How much water was left in Imani's gallon container?

### New Concept

Sometimes we need to add whole numbers and decimal numbers in the same problem. Read the example on the next page.



The Simpsons hired a carpenter to cut an opening in their wall and to install a new door. The carpenter needed to order a frame for the door to cover the thickness of the wall. The carpenter knew that the siding was 1 inch thick, that the wall stud was 3.5 inches thick, and that the drywall was 0.5 inches thick.



To find the thickness of the wall, the carpenter writes 1 inch as 1.0 inches, aligns the decimal points of all three measurements, and adds. He finds that a 5.0-inch-thick door frame is needed.

$$\begin{array}{r} 1.0 \text{ in.} \\ 3.5 \text{ in.} \\ + 0.5 \text{ in.} \\ \hline 5.0 \text{ in.} \end{array}$$

### Thinking Skill

#### Verify

Can we write zeros after a whole number without changing the value of the number? Explain.

The carpenter wrote the whole number 1 as the decimal number 1.0 so that he could align the decimal points before adding. Since a decimal point marks the end of a whole number, we may add a decimal point to the back (right-hand side) of a whole number. After placing the decimal point, we may also attach zeros to make arithmetic with the whole number easier.

### Example 1

**In a science experiment, the scientist placed food at the end of a short maze and placed a mouse at the beginning of the maze. The scientist timed how long it took for the mouse to reach the food. Then the scientist repeated the experiment three times. The mouse's times were 6.2 seconds, 4.25 seconds, and 3 seconds. What was the total elapsed time of the three trials?**

We find the total time by adding. To add digits with the same place value, we align decimal points. In this problem the whole number 3 has the same place value as the 6 and the 4. We place a decimal point to the right of the 3 and align decimal points. We may choose to fill empty decimal places with zeros. The total elapsed time is **13.45 sec.**

$$\begin{array}{r} 6.20 \\ 4.25 \\ + 3.00 \\ \hline 13.45 \end{array}$$

**Connect** Name the place value of each digit in the sum.

## Example 2

A computer that began a complex task 8 minutes ago needs 24.6 minutes altogether to complete the task. In how many minutes will the task be complete?

To find the remaining time, we subtract.

We place a decimal point to the right of the whole number 8 and then align decimal points before subtracting.

We may fill the empty decimal place with a zero if we wish. The task will be completed in **16.6 minutes**.

$$\begin{array}{r} 24.6 \\ - 8.0 \\ \hline 16.6 \end{array}$$

**Represent** Write the difference using words.

## Example 3

Which digit in 4.65 is in the same place as the 2 in 12?

The 2 in 12 is in the ones place. In 4.65 a decimal point separates the ones place and the tenths place, marking the end of the whole number and the beginning of the fraction. So the **4** in 4.65 is in the same place as the 2 in 12.

## Lesson Practice

Find each sum or difference:

a.  $4.3 + 2$

b.  $12 + 1.2$

c.  $6.4 + 24$

d.  $4 + 1.3 + 0.6$

e.  $5.2 + 0.75 + 2$

f.  $56 + 75.4$

g.  $8 + 4.7 + 12.1$

h.  $9 + 4.8 + 12$

i.  $4.75 - 2$

j.  $12.4 - 5$

k. Which digit in 24.7 is in the same place as the 6 in 16?

l. Compare:  $12 \bigcirc 12.0$

## Written Practice

*Distributed and Integrated*

- <sup>(97)</sup> \*1. There were 50 boys and 60 girls on the playground. What was the ratio of girls to boys on the playground?
- <sup>(71, 81)</sup> 2. The apple was sliced into 6 equal pieces. Brayden ate 2 pieces. What fraction of the apple did he eat? What percent of the apple did he eat?

3. Pens were on sale. Wayne bought five of them for \$1. At this rate, what would be the price for a dozen pens?

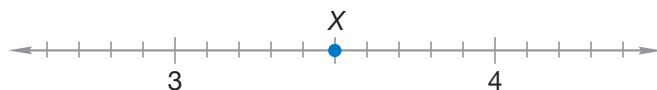
(49)

\*4. **Analyze** Alexandra ran 100 yards in 13.8 seconds. Owen ran 1 second slower than Alexandra. How long did it take Owen to run 100 yards?

(35, 99)

\*5. **Connect** Name point X on the number line below as both a decimal number and a reduced mixed number.

(66, 81)



\*6. If  $10n = 100$ , then  $n^2$  equals what number?

(78)

\*7. **Represent** Write the decimal number one thousand, six hundred twenty and three tenths.

(68)

8. A rectangle measures 6 ft 10 in. tall and 2 ft 11 in. wide. Estimate the area of the rectangle.

(62, 72)

9. **Analyze** Write a fraction equal to  $\frac{3}{4}$  that has a denominator of 8. Then subtract that fraction from  $\frac{7}{8}$ .

(79)

10. Is \$7.13 closer to \$7 or \$8?

(69)

11.  $QT$  is 100 mm.  $QR$  is 23 mm.  $RS$  equals  $QR$ . Find  $ST$ .

(61)



\*12.  $3.4 + 5$

(99)

\*13.  $7.25 - 7$

(99)

14.  $\sqrt{25} - \sqrt{16}$

(78)

15.  $60^2$

(78)

\*16.  $28\overline{)952}$

(94)

17.  $\$18.27 \div 9$

(34)

\*18.  $4\frac{5}{8} + 1\frac{7}{8}$

(91)

19.  $5 - \left(2\frac{3}{5} - 1\right)$

(43, 63)

\*20.  $\frac{3}{4} \times \frac{1}{3}$

(90)

\*21.  $\frac{3}{4} \div 3$

(96)

22.  $\frac{9}{10} = \frac{\square}{100}$


(79)

23. Reduce:  $\frac{20}{100}$

(90)

24. Use the table below to answer parts **a** and **b**.  
(Inv. 4)

<b>Gallons</b>	1	2	3	4	5
<b>Fluid Ounces</b>	128	256	384	512	640

- a.  **Generalize** Write a rule that describes how to find the number of fluid ounces for any number of gallons.
- b. **Predict** How many fluid ounces is 8 gallons?
- \*25. A low temperature in Ely, Minnesota, was ten degrees below zero Fahrenheit. Write that temperature using digits and symbols.  
(98)
26. **Conclude** There is a pattern to the differences between successive terms of this sequence:  
(Inv. 4)

3, 4, 7, 12, 19, ...

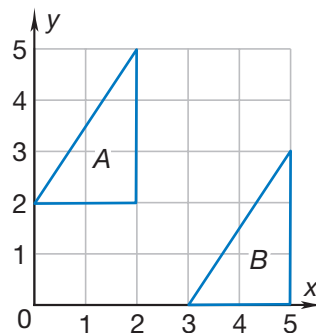
Assuming the pattern of differences continues, find the next three terms of the sequence.

27. A standard number cube is rolled once.  
(Inv. 9)
- a. What is the probability that the upturned face is 3 or less?
- b. Describe a different event that has the same probability.
- \*28. Write two billion, six hundred million in expanded notation using powers of 10.  
(78)

- \*29. **Multiple Choice** Which transformation would position triangle A on triangle B?  
(Inv. 8)

**A** translation  
**C** reflection

**B** rotation  
**D** flip



30. In an election for class president,  $\frac{3}{8}$  of the students voted for KaMaria and  $\frac{3}{8}$  of the students voted for Ashley. In simplest form, what fraction of the students voted for KaMaria or Ashley?  
(41)

## • Simplifying Decimal Numbers

### Power Up

#### facts

#### mental math

#### problem solving

#### Power Up I

- a. **Number Sense:** Simplify the improper fractions  $\frac{15}{10}$ ,  $\frac{20}{10}$ , and  $\frac{25}{10}$ .
- b. **Number Sense:** What is the reciprocal of 3? ... of  $\frac{3}{5}$ ?
- c. **Percent:** What is 25% of \$100? ... 25% of \$10? ... 25% of \$1?
- d. **Time:** How many years are in two and a half centuries?
- e. **Time:** How many years are in two and a half millennia?
- f. **Powers/Roots:**  $5^2 - 1^3$
- g. **Measurement:** The topmost shelf is 2 yards above the floor. How many inches is that?
- h. **Roman Numerals:** Write 27 in Roman numerals.

Choose an appropriate problem-solving strategy to solve this problem. Cassandra likes to write letters to pen pals. She writes 6 letters each month. The envelopes Cassandra uses to send her letters are packaged 45 per box. How many boxes of envelopes must Cassandra purchase to last one year? How many envelopes will she have left over at the end of one year? Explain how you solved the problem.

## New Concept

When we write numbers, we usually write them in simplest form. To simplify a number, we change the form of the number, but we do not change the value of the number. For example, we simplify fractions by reducing. Often, we can simplify decimal numbers by removing unnecessary zeros. We will explain this by simplifying 0.20.

The decimal number 0.20 has a 2 in the tenths place and a 0 in the hundredths place. The zero in the hundredths place means “no hundredths.” If we remove this zero from 0.20, we get 0.2. The number 0.2 also has a 2 in the tenths place and “no hundredths.” Thus, 0.20 equals 0.2. We say that 0.20 simplifies to 0.2.

### Thinking Skill

#### Discuss

Why are 0.200 and 0.2 different names for the same number?

**We can remove zeros from the front of whole numbers and from the back of decimal numbers.** We may remove zeros until we come to a digit that is not a zero or until we come to a decimal point. Below we have simplified 02.0100, 20.0, and 0.200 by removing unnecessary zeros.

02.0100	20.0	0.200
Simplified → 2.01	20	0.2 or .2

In the center example, we took two steps to simplify 20.0. After removing the unnecessary zero, we also removed the decimal point. **A decimal point can be removed when there is no fraction part to a number.**

### Reading Math

If a decimal such as .2 is written without a zero in the ones place, the decimal point may not be noticed. We write a zero in front of a decimal number that is less than 1 to ensure that the decimal point is noticed.

To simplify 0.200, we removed the two trailing zeros, leaving 0.2 as the simplified form. We can also remove the zero in front, leaving .2 as the simplified form. The numbers 0.2 and .2 are equal, and both forms are correct. However, if the whole-number part of a decimal number is zero, it is customary to write a zero in the ones place, which is what we will do in this book. Note that calculators also display a zero in the ones place of such numbers.

In some situations we might want to attach zeros to a decimal number. The decimal point of a decimal number determines place value, not the number of digits. So attaching zeros at the end of a decimal number does not change the place values.

### Example 1

Otis added 3.75 to 2.75 and found that the sum was 6.50. Simplify the sum.

We may remove the ending zero(s) of a decimal number.

$$6.50 = \mathbf{6.5}$$

### Example 2

Attach a zero to the end of 5 without changing the value of the number.

If we attach a zero to 5 without using a decimal point, we get 50, which does not equal 5. So we write the whole number 5 with a decimal point and then attach a zero.

$$5 = \mathbf{5.0}$$

### Lesson Practice

Simplify each decimal number:

- a. 03.20      b. 0.320      c. 32.00      d. 3.020

Simplify each answer:

- e.  $\begin{array}{r} 3.65 \\ + 6.35 \\ \hline \end{array}$       f.  $\begin{array}{r} 23.16 \\ - 19.46 \\ \hline \end{array}$       g.  $\begin{array}{r} 4.23 \\ - 3.18 \\ \hline \end{array}$

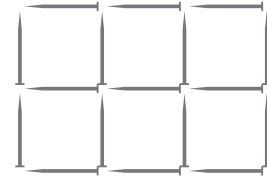
- h. Attach a zero to the end of 2.5 without changing its value.  
i. Attach a zero to the end of 6 without changing its value.

## Written Practice

*Distributed and Integrated*

- \*1. <sup>(97)</sup> Cody counted 60 peas and 20 carrot slices on his plate. What was the ratio of carrot slices to peas on his plate?
2. <sup>(49)</sup> A package of 10 rolls costs \$3.25. At that price, what would be the cost of 100 rolls?
3. <sup>(46, 71)</sup> Three fourths of the 28 students finished the test early. How many students finished the test early? What percent of the students finished the test early?

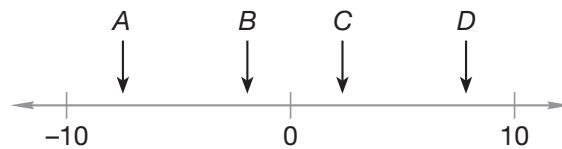
4. The rectangle was formed with pins 1 inch long.



- How many pins form the perimeter?
- How many small squares cover the rectangle?

- \*5. Attach a zero to the end of 8 without changing the value of the number.

- \*6. a. **Connect** Which arrow could be pointing to  $7\frac{3}{4}$  on the number line below?

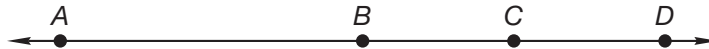


- b. Which arrow could be pointing to negative 2?

- \*7. **Analyze** Write fractions equal to  $\frac{5}{6}$  and  $\frac{3}{4}$  that have denominators of 12. Then subtract the smaller fraction from the larger fraction.

- \*8. The giraffe stood 5 meters tall. Five meters is how many centimeters?

9.  $AB$  is 40 mm.  $BC$  is half of  $AB$ .  $CD$  equals  $BC$ . Find  $AD$ .



\*10.  $6.2 + 3 + 4.25$

11.  $10^3 - 10^2$

\*12.  $6.37 - 6$

13.  $234 \times 506$

\*14.  $10 \times \$1.75$

15.  $\$17.50 \div 10$

16.  $\frac{1}{50} = \frac{\square}{100}$

17. Reduce:  $\frac{40}{100}$

18.  $\sqrt{64}$

19.  $16w = 832$

\*20.  $\frac{5}{9} + \frac{5}{9} + \frac{5}{9}$

21.  $\frac{9}{10} \times \frac{9}{10}$

\*22.  $\frac{2}{3} \div \frac{3}{4}$

\*23.  $3 \div \frac{3}{4}$

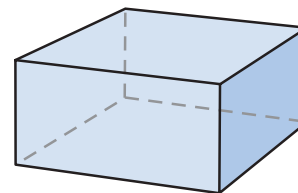
24. The flagpole is 10 yards tall. The flagpole is how many feet tall?



- \*25. <sup>(28)</sup> In most years, the first day of summer begins on June 21 and the first day of winter begins on December 21. During those years, what is the elapsed time in days from the first day of summer to the first day of winter?

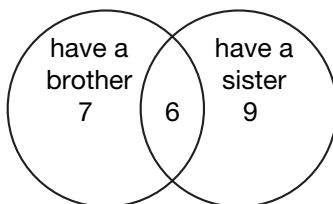
26. <sup>(83)</sup> A rectangular prism has

- how many faces?
- how many edges?




- \*27. <sup>(98)</sup> Water freezes at  $0^{\circ}\text{C}$ . What temperature on the Celsius scale is five degrees colder than the freezing temperature of water?

- \*28. <sup>(Inv. 7)</sup> Thirty children were asked whether they have a sister and whether they have a brother. The Venn diagram below records their responses. Use this information to answer parts **a–c**.



- How many children have a brother?
  - How many have a sister but not a brother?
  - The numbers in the circles do not add up to 30. What does that mean?
- \*29. <sup>(Inv. 6)</sup> The data below describe the first 30 minutes of the flight of a homing pigeon during a 100-mile flight:

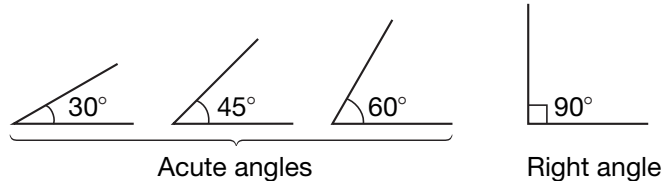
<b>Elapsed Time (in minutes)</b>	0	10	20	30
<b>Distance Traveled (in miles)</b>	0	6	14	22

- Display the data in a line graph.
  -  **Predict** About how long will it take the pigeon to complete the flight? Explain your answer.
- \*30. <sup>(49)</sup> The length of the Wisconsin River is 30 miles longer than one half the length of the North Canadian River in New Mexico and Oklahoma. The North Canadian River is 800 miles long. What is the length of the Wisconsin River?

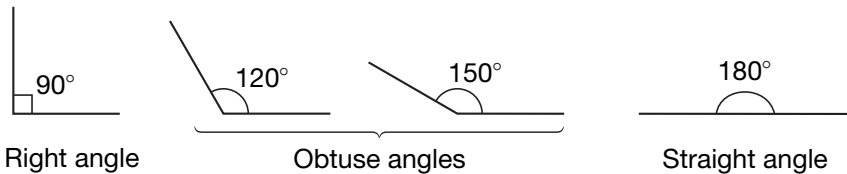
Focus on

• Measuring Angles

One way to measure an angle is with **degrees**. Here we show four angles and their measures in degrees:



Note that a right angle measures 90°, and that acute angles measure less than 90°. Obtuse angles measure more than 90° and less than a straight angle, which measures 180°.

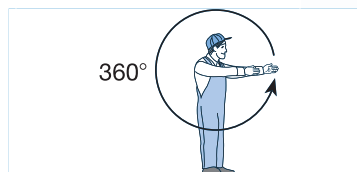
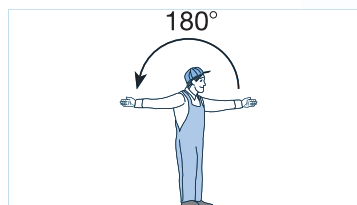
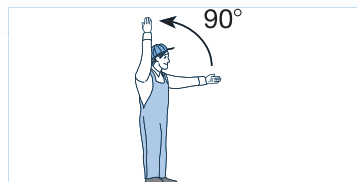


A full circle contains 360°, as demonstrated in the activity below.

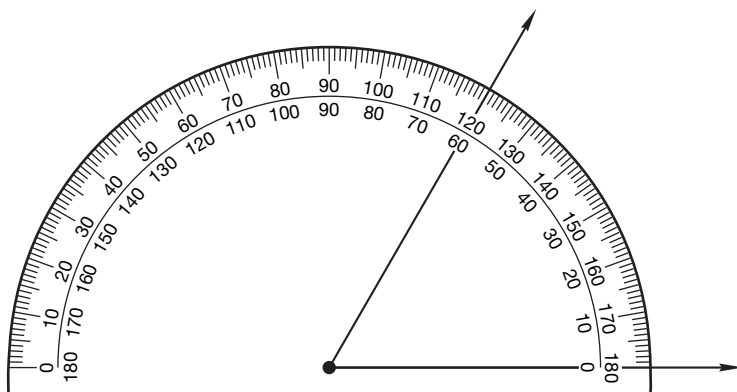
Activity 1

Modeling Angles

1. Beginning with your arms extended forward at 0°, raise one arm to form a 90° angle.
2. Beginning with your arms extended forward at 0°, raise one arm up, around, and halfway down to form a 180° angle.
3. Beginning with your arms extended forward at 0°, move one arm up through 90°, down through 180°, and continue around to 360°.

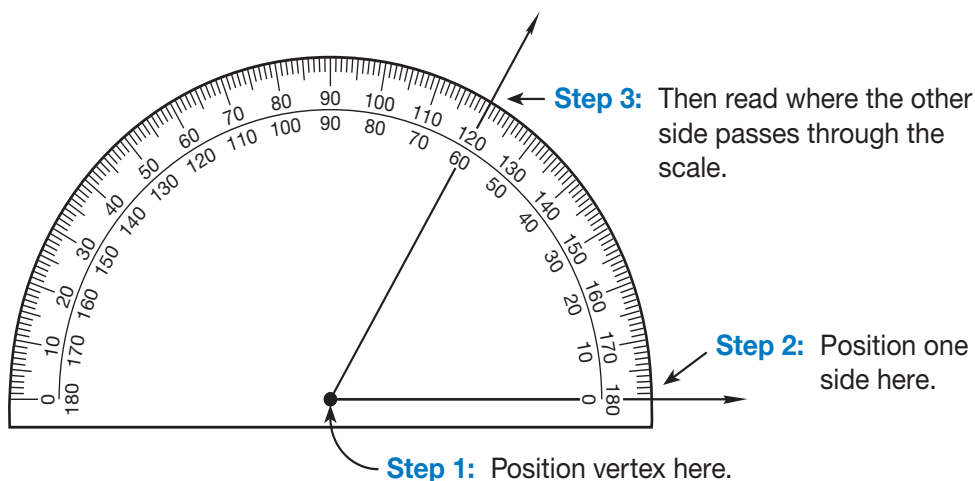


A **protractor** is a tool for measuring angles. A typical protractor like the one below usually has two scales: one ranging from  $0^\circ$  to  $180^\circ$  from left to right, and the other ranging from  $0^\circ$  to  $180^\circ$  from right to left. By paying attention to whether you are measuring an acute angle or an obtuse angle, you will know which scale to read. The angle being measured below is an acute angle, so we know that its measure is  $60^\circ$  and not  $120^\circ$ .



To measure an angle, we use three steps. (Refer to the illustration below.)

- Step 1:** Position the center of the protractor curve on the vertex of the angle.
- Step 2:** Also position one of the  $0^\circ$  marks on one side of the angle.
- Step 3:** Check that both Steps 1 and 2 have been done correctly; then read the scale where the other side of the angle passes through the scale.



## Activity 2

### Measuring Angles

Materials needed:

- Lesson Activity 42
- protractor

4. With your protractor, measure each angle on **Lesson Activity 42**.

## Activity 3

### Drawing Angles

Materials needed:

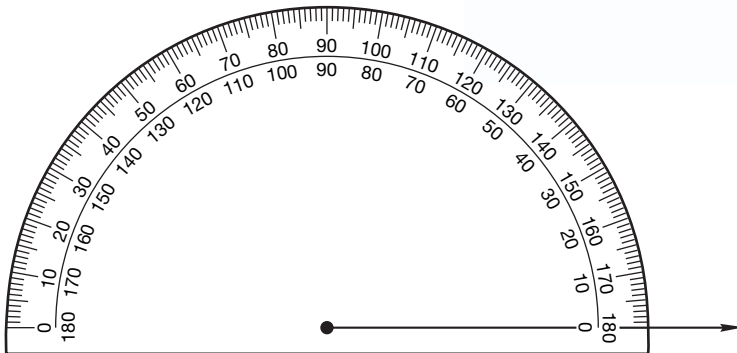
- protractor
- unlined paper

We can use a protractor to help us draw an angle of a specific size. Follow these steps:

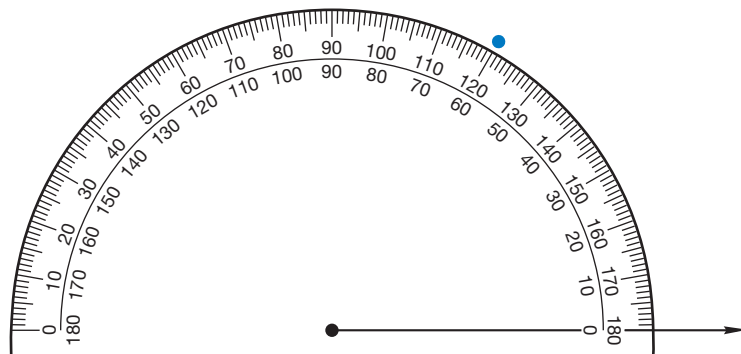
**Step 1:** Using the straight edge of your protractor, draw a segment long enough to extend from the center of the protractor through the scale.



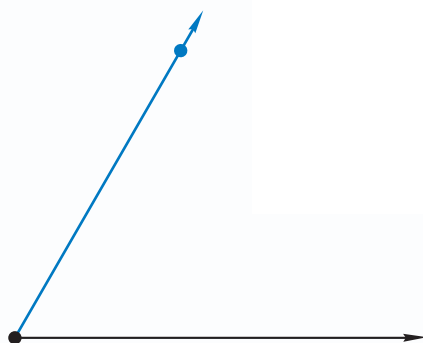
**Step 2:** Position the protractor so that the center is over one endpoint of the segment (the intended vertex) and that the segment passes through a  $0^\circ$  mark.



**Step 3:** Find the number on the protractor that matches the size of the angle you wish to draw. (Be sure you are reading from the correct scale.) Make a dot on your paper even with the scale mark on the protractor. We have shown a mark for a  $60^\circ$  angle.



**Step 4:** Remove the protractor and draw the remaining side of the angle from the vertex through the dot.



Follow Steps 1–4 from Activity 3 to draw angles with these measures:

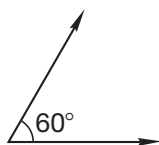
5.  $30^\circ$

6.  $90^\circ$

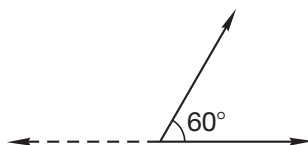
7.  $110^\circ$

8.  $70^\circ$

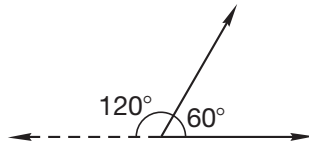
Angles have interior and exterior measurements. In Activity 3, you drew an angle with an interior measurement of  $60^\circ$ .



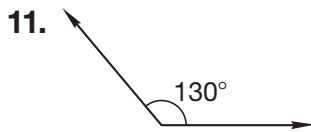
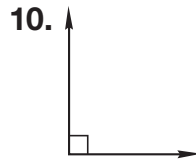
We can also measure exterior angles by extending one of the rays to make a line.



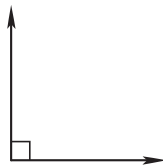
The exterior angle can be measured in two ways: using a protractor or subtracting the interior angle from  $180^\circ$ . We can see that both angles make one straight angle ( $180^\circ$ ). By subtracting  $60^\circ$  from  $180^\circ$ , we find that our exterior angle is  $120^\circ$ .



Find the exterior angle measurement for the angles in problems **9–11**.



- a.** Draw a diagram to show this angle rotated  $90^\circ$  clockwise and  $90^\circ$  counterclockwise. Which of the two rotations produces the same image as a reflection of the given angle across its vertical side?



- b.** A reflex angle is an angle greater than  $180^\circ$  and less than  $360^\circ$ . Use your protractor to draw a reflex angle. Explain your reasoning.